The Nobel Prize in Physics 2007: Giant Magnetoresistance

An idiosyncratic survey of Spintronics from 1963 to the present

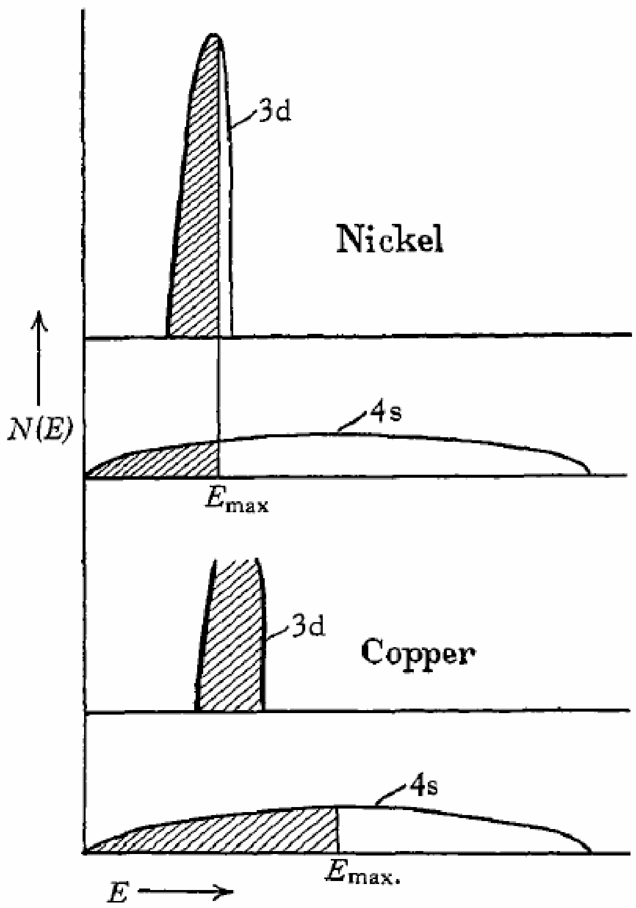
The early years

1935-85

The s-d model

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Mott, Proc. Phys. Soc. **47**, 571 (1935)



The positive holes in the d band make a certain contribution to the conductivity-i.e. they are free to move through the lattice. But since the atomic d wave functions do not overlap much, the positive holes will take a much longer time to move from one atom to the next than would be taken by an s electron, and so the contribution to the effective number of free electrons is small. The positive holes, however,will increase the ***resistance*** in the following way.

The resistance of a metal is proportional, among other things, to the number of times per second an electron is scattered, i.e., to the number of times per second that it makes a transition from a state specified by a wave vector k to any other state k'. Now the probability for such a transition is proportional to N (E) the density of states ; for if N (E) is big, there are more states into which the electron can jump. In the transition metals, N (E) is **big** in the d band; and therefore electrons will jump more frequently from the s to the d band than from one s state to another. The time of relaxation for such metals is therefore shorter, and the conductivity smaller than for the noble metals, in which only s-s transitions can take place.

A year later Mott [Proc. Roy. Soc. A153, 699(1936)] further develops his ideas on the temperature dependence of the conductivity of the transition-metals Mott

It was shown from an examination of the experimental evidence that the conduction electrons in these metals have wave functions derived mainly from s states just as in Cu, Ag, and Au, and that the effective number of conduction electrons is not much less than in the noble metals. On the other hand, **the mean free path is much smaller**, because under the influence of the lattice vibrations the conduction electrons may make transitions to the unoccupied d states, and the probability of these transitions is several times greater than the probability of ordinary scattering. *Since the unoccupied d states are responsible for the ferromagnetism or high paramagnetism of the transition elements, there is a direct connexion between the magnetic properties and the electrical conductivity*. Editorial comment: this is **magnetoresistance.**

Paris 1968-76 Fert & Campbell
Two current model of conduction in ferromagnetic metals

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

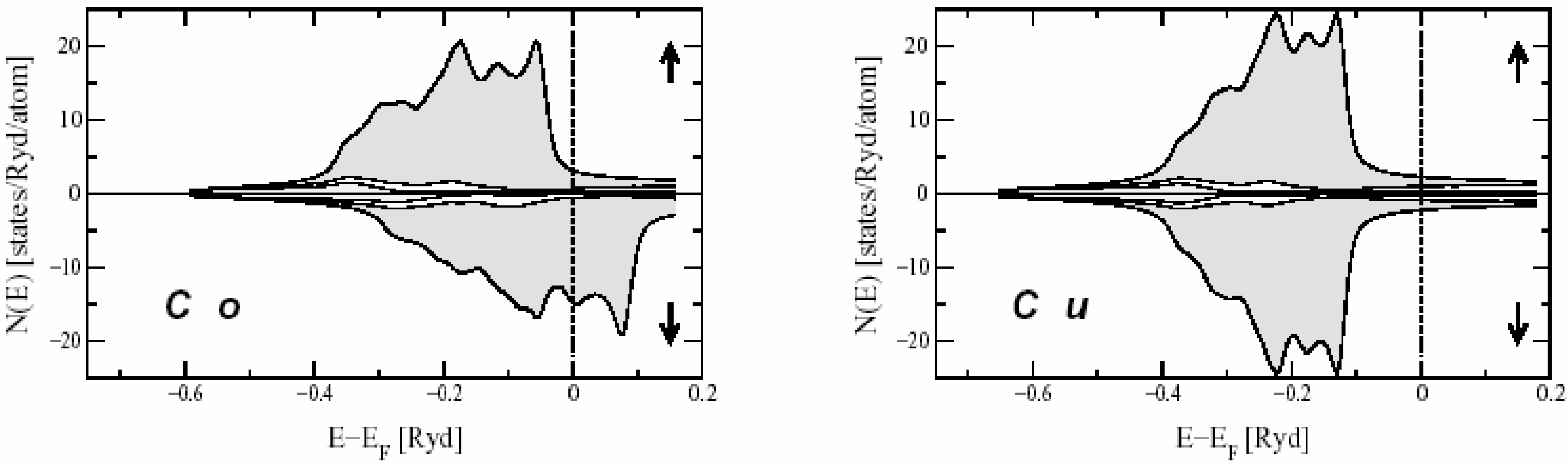


Figure 2.6: Spin-projected densities of states for Co and Cu; the differently shaded grey areas indicate the amount of s, p and d states.

A key observation of Fert from his studies (1967-73) of the two current model was that certain impurities increased the resistivity of metallic alloys far more than others. This is summarized in Fert & Campbell, J.Phys. F:Metal Phys. 6, 849 (1976).

Sec. 4. *Residual resistivity of ternary dilute alloys*

"…we have measured the residual resistivity of ternary alloys and observed clear deviations from MR which are shown on figure 1 for **Ni**VCo, **Ni**VFe, **Ni**CrMn and **Ni**CrTi alloys. … We note that most impurities can be separated into a first group (Co,Fe,Mn) with very high values of [alpha] and a second group (Cr,V) for which this is smaller than one. This explains the large deviations from MR for ternary alloys containing an element of each group (e.g. **Ni**CoV). On the contrary the deviations are negligible when both impurities have nearly the same ratio-alpha…"

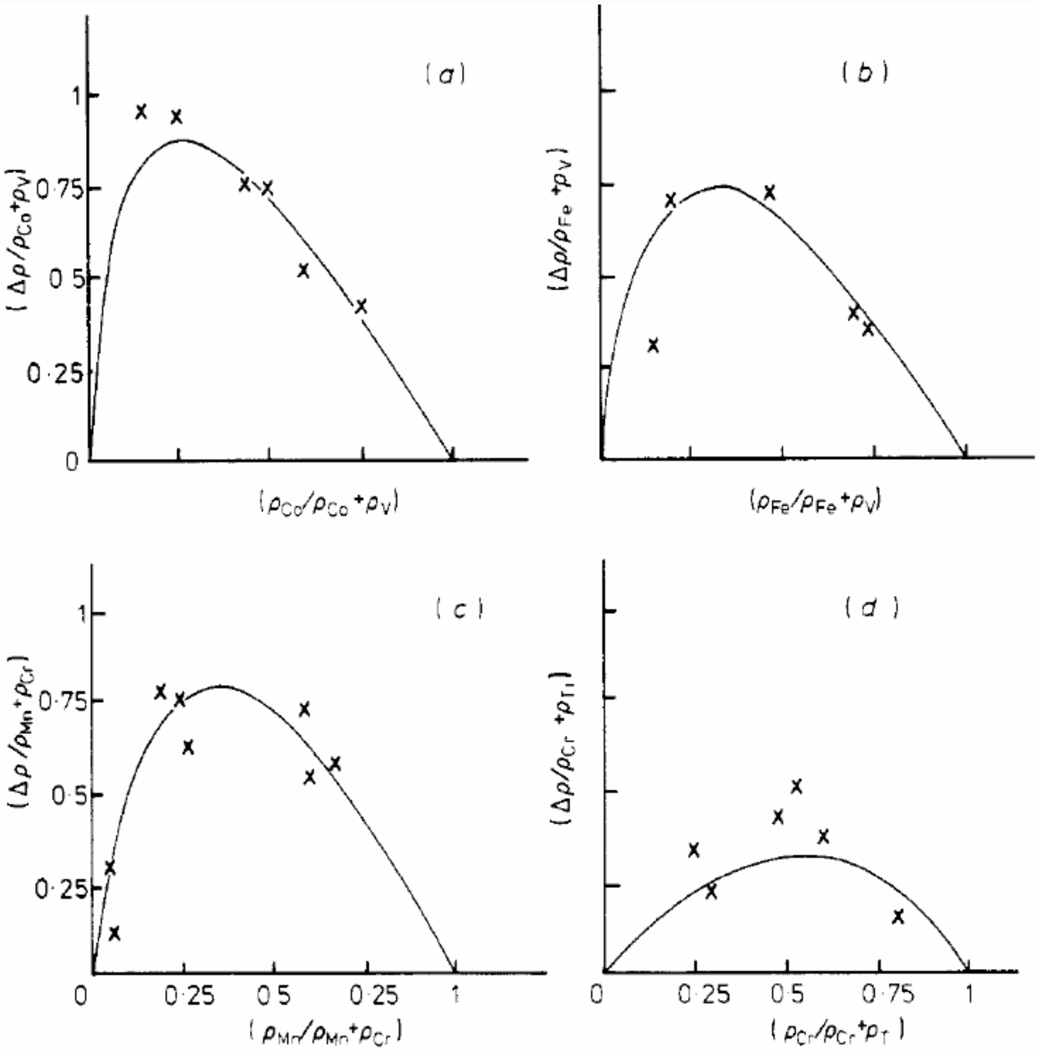


Figure 1. Deviations from Matthiessen's rule in Ni based ternary alloys (relative deviations against relative resistivities). (*a*), **Ni**CoV; (*b*), **Ni**FeV; (*c*), **Ni**MnCr; (*d*), **Ni**CrTi. The full curves are calculated from expression (19) with the values of α listed in table 1.

Table 1. The residual resistivity per at%, the parameter $\alpha = \rho_{0\downarrow}/\rho_{0\uparrow}$ and the spin ↑ and spin ↓ residual resistivities per at% for 3d impurities in nickel.

Impurity	Ti	V	Cr	Mn	Fe	Co
Resistivity ρ_0 (μΩ cm/at%)	2·9	4·5	5·0	0·61	0·35	0·145
$\alpha = \rho_{0\downarrow}/\rho_{0\uparrow}$	4 $(3 < \alpha < 5)$	0·55 $(0{\cdot}5 < \alpha < 0{\cdot}6)$	0·45 $(0{\cdot}35 < \alpha < 0{\cdot}5)$	15 $(11{\cdot}5 < \alpha < 17)$	20 $(15 < \alpha < 23)$	30 $(23 < \alpha < 33)$
$\rho_{0\uparrow}$ (μΩ cm/at%)	3·6	12·7	16·1	0·65	0·37	0·15
$\rho_{0\downarrow}$ (μΩ cm/at%)	14·5	7·0	7·2	9·8	7·4	4·6

In this table we discern the origins of the idea of Giant Magnetorsistance

Fert's conclusion:

If the magnetic moment of an impurities is antiparallel to the host magnetization, or if the moments of ternary impurities are antiparallel, the resistivity is higher than when they are parallel.

The question that remained was:

How can one switch the moments from parallel to antiparallel ?

The answer:

Use multilayered structures which allowed one to rotate the magnetization of one magnetic layer relative to another.

Grenoble 1963 Néel et al.
Interlayer coupling

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Proceedings of ICM’64

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

The early period: 1960-85

- Heterostructures 1970's Esaki
- Metallic multilayers- 1980's Schuller, Shinjo, Prinz,Grünberg

Metallic multilayers
&
GMR

1985-95

1985 - 1995

- Interlayer coupling 1986, Grünberg, Salamon,Flynn, Kwo, Majkrzak, Yafet.

- Spin accumulation and injection -1987 Silsbee and Johnson von Son and Wyder

- Giant magnetoresistance [GMR] 1988
- Fert and Grünberg

VOLUME 61, NUMBER 21 PHYSICAL REVIEW LETTERS 21 NOVEMBER 1988

Giant Magnetoresistance of (001) Fe/(001) Cr Magnetic Superlattices

M. N. Baibich,[a] J. M. Broto, A. Fert, F. Nguyen Van Dau, and F. Petroff
Laboratoire de Physique des Solides, Université Paris-Sud, F-91405 Orsay, France

P. Eitenne, G. Creuzet, A. Friederich, and J. Chazelas
Labora ... Orsay, France

No.6 of the most cited PRL’s

We have studied the magnetoresistance of (001)Fe/(001)Cr superlattices prepared by molecular-beam epitaxy. A huge magnetoresistance is found in superlattices with thin Cr layers: For example, with $t_{Cr}=9$ Å, at $T=4.2$ K, the resistivity is lowered by almost a factor of 2 in a magnetic field of 2 T. We ascribe this giant magnetoresistance to spin-dependent transmission of the conduction electrons between Fe layers through Cr layers.

Data on GMR

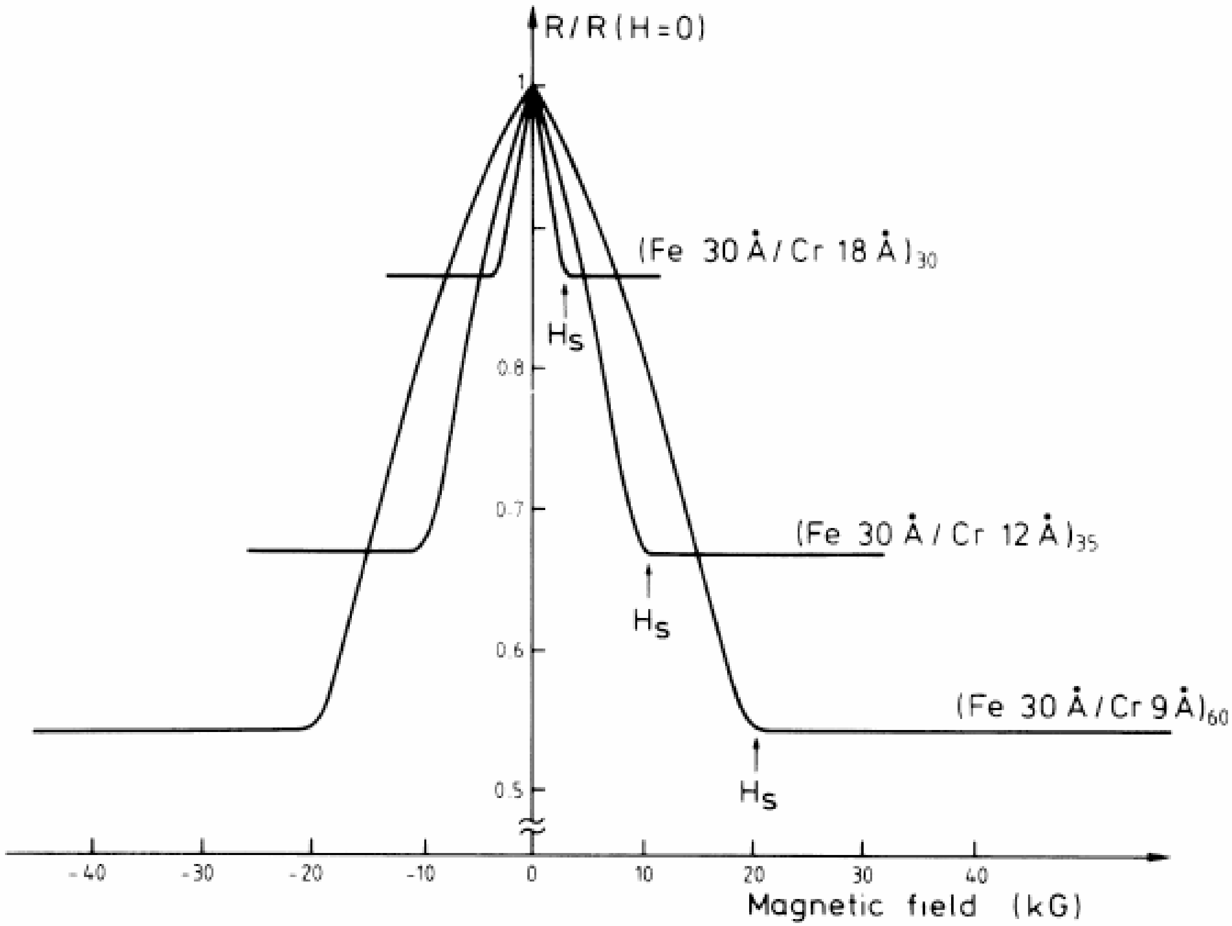
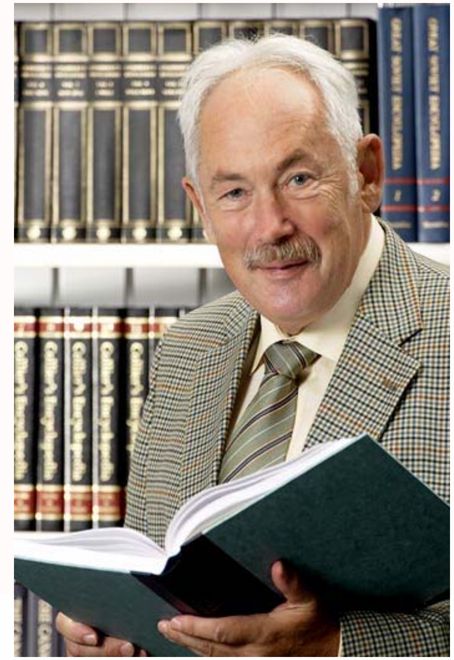


FIG. 3 Magnetoresistance of three Fe/Cr superlattices at 4.2 K. The current and the applied field are along the same [110] axis in the plane of the layers.

M.N. Baibich *et al.*, Phys. Rev. Lett. **61**, 2472 (1988).



RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 39, NUMBER 7 1 MARCH 1989

Enhanced magnetoresistance in layered magnetic structures with antiferromagnetic interlayer exchange

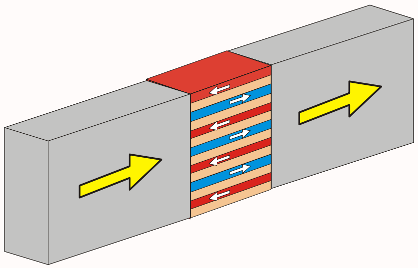
G. Binasch, P. Grünberg, F. Saurenbach, and W. Zinn
Institut für Festkörperforschung, Kernforschungsanlage Jülich G.m.b.H., Postfach 1913, D-5170 Jülich, West Germany
(Received 31 May 1988; revised manuscript received 12 December 1988)

The electrical resistivity of Fe-Cr-Fe layers with antiferromagnetic interlayer exchange increases when the magnetizations of the Fe layers are aligned antiparallel. The effect is much stronger than the usual anisotropic magnetoresistance and further increases in structures with more than two Fe layers. It can be explained in terms of spin-flip scattering of conduction electrons caused by the antiparallel alignment of the magnetization.

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

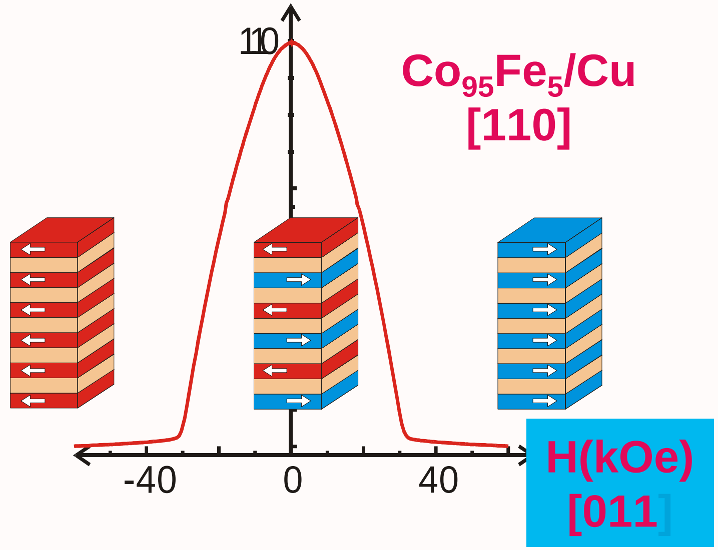
- Spin valves 1992, Speriosu, Dieny, Parkin

GMR in Multilayers and Spin-Valves

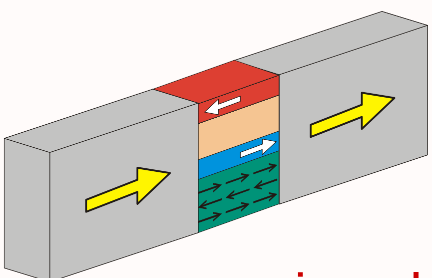


multi-layer

ΔR/R~110% at RT
Field ~10,000 Oe

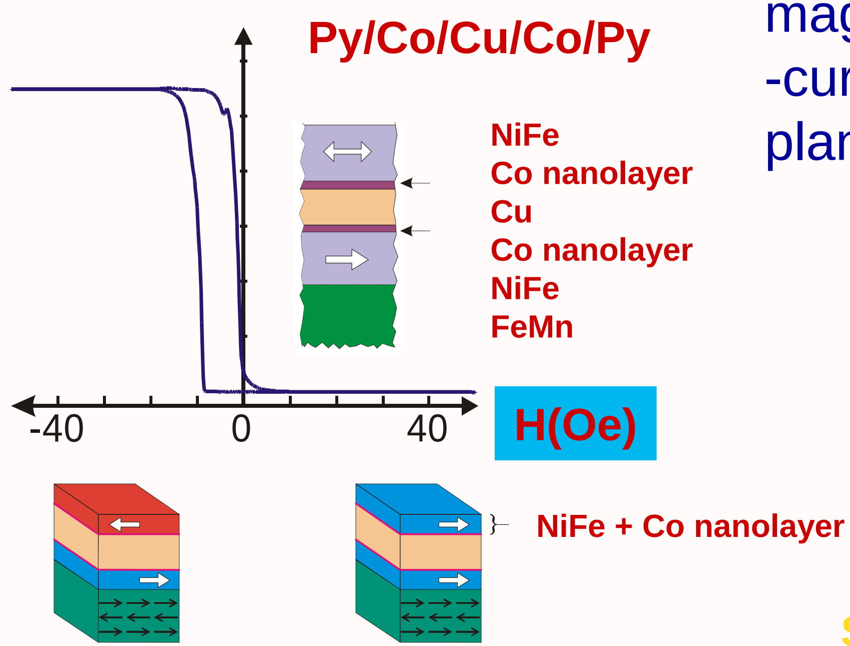


GMR
-metallic spacer between magnetic layers
-current flows in-plane of layers



spin-valve

ΔR/R~8-17% at RT
Field ~1 Oe



S.S.P. Parkin

1995 GMR heads

QuickTime?and a TIFF (Uncompressed) decompressor are needed to see this picture.

From IBM website

Spintronics- control of current through spin of electron

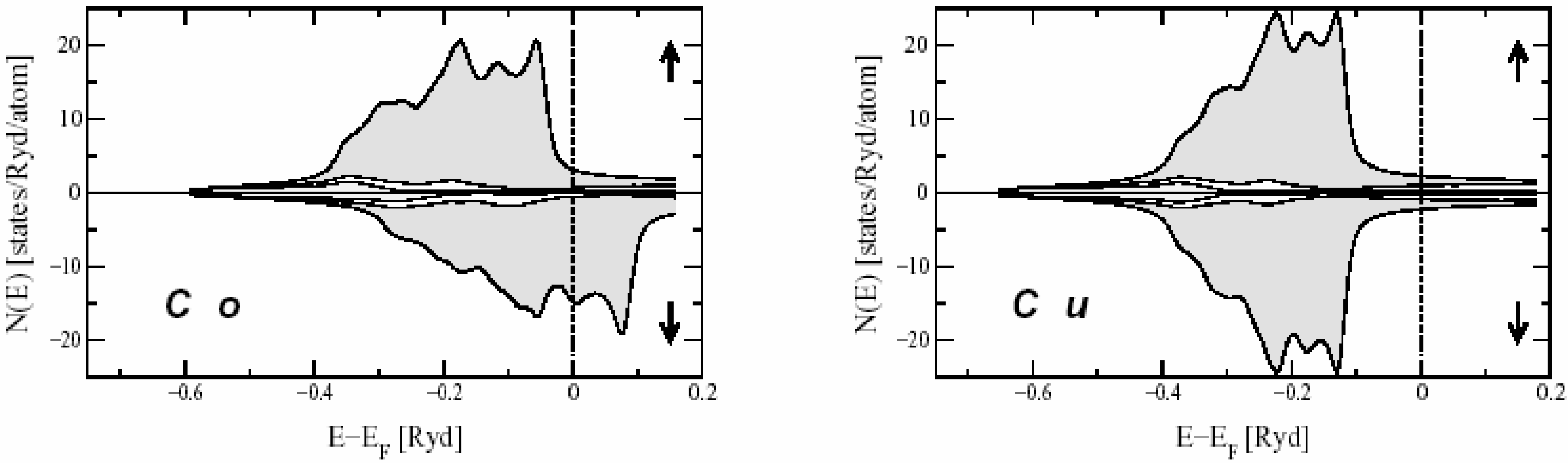


Figure 2.6: Spin-projected densities of states for Co and Cu; the differently shaded grey areas indicate the amount of s, p and d states.

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

The two current model of conduction in ferromagnetic metals

spin. When we exclude spin-flip scattering processes and make the gross yet conventional assumption that all k states for a spin direction scatter at the same rate we arrive at a particularly simple parameterization of conduction in which one assigns a scattering rate or resistivity to each spin channel of conduction (see also Eq. 2.30):

$$\rho^{\uparrow,\downarrow} = \rho^{M,m} \equiv a \pm b, \tag{2.77}$$

where the superscripts M and m, refer to electrons with spin parallel (Majority) and opposite (minority) to the magnetization. From Eq. (2.65) and the realization that for a homogeneous sample the effective fields are independent of spin, the total current in the two spin channels is,

$$j = j^{\uparrow} + j^{\downarrow} = (\sigma^{\uparrow} + \sigma^{\downarrow})E, \tag{2.78}$$

$$\rho^{\uparrow,\downarrow} = \rho^{M,m} \equiv a \pm b,$$

Resistors in series

$$\rho = \rho^{\uparrow} + \rho^{\downarrow} = 2a$$

Resistors in parallel

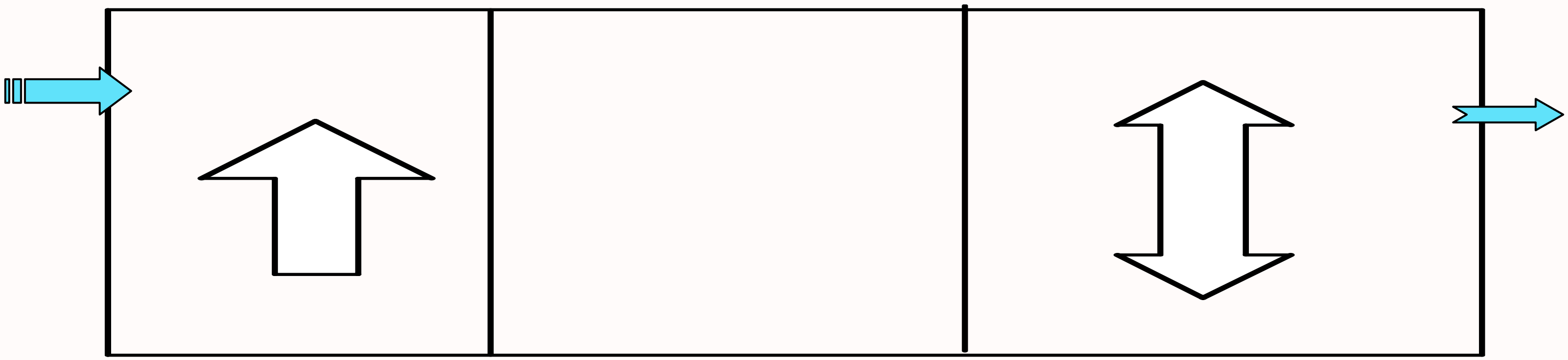
$$j = j^{\uparrow} + j^{\downarrow} = (\sigma^{\uparrow} + \sigma^{\downarrow})E,$$

$$\rho = \frac{1}{\sigma^{\uparrow} + \sigma^{\downarrow}} = \frac{\rho^{\uparrow}\rho^{\downarrow}}{\rho^{\uparrow} + \rho^{\downarrow}}$$

$$= \frac{a^2 - b^2}{2a}.$$

1988 Giant magnetoresistance

Albert Fert & Peter Grünberg



Parallel configuration

Antiparallel configuration

Two current model in magnetic multilayers

$$\rho^{\uparrow} = \frac{1}{4}\left[2\rho^{M} + 2\rho^{n}\right],$$

$$\rho^{\downarrow} = \frac{1}{4}\left[2\rho^{m} + 2\rho^{n}\right],$$

$$\rho^{\uparrow} = \rho^{\downarrow} = \frac{1}{4}\left[\rho^{M} + \rho^{m} + 2\rho^{n}\right],$$

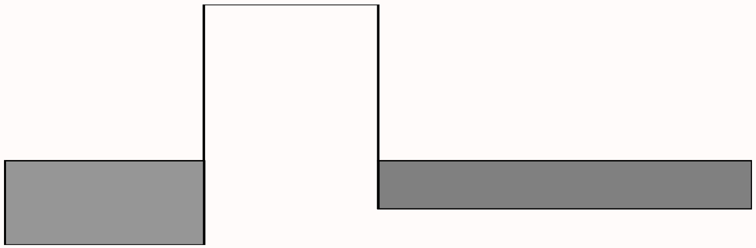
$$\rho_P = \frac{1}{4}\left\{(a+c) - \frac{b^2}{(a+c)}\right\},$$

$$\rho_{AP} = \frac{1}{4}(a+c).$$

Magnetic tunnel junctions
&
MRAM

1995-2000

Tunneling-MR



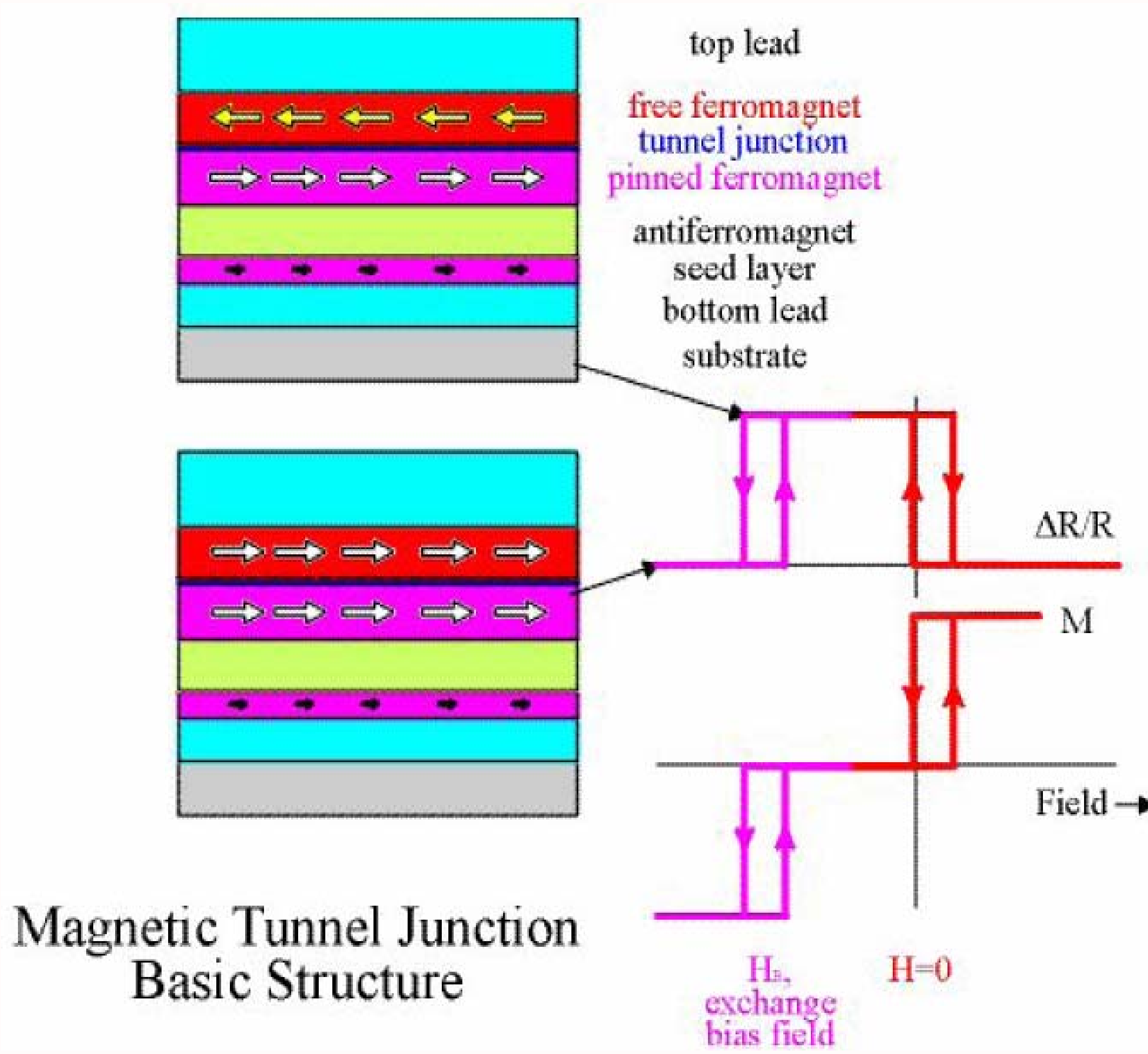
Two magnetic metallic electrodes separated by an insulator; transport controlled by tunneling phenomena not by characteristics of conduction in metallic electrodes

• Spin currents in tunnel junctions 1989, Slonczewski

2000 magnetic tunnel junctions used in magnetic random access memory

From IBM website;

http://www.research.ibm.com/research/gmr.html



1995-2000

- Reproducible MR with MTJ’s 1995, Moodera, Meservey, and Miyazake

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

1995-2000

- Magnetic Random Access Memory 1997 DARPA Initiative {IBM, Motorola, Honywell, NYU}

1995-2000

- Crystalline barriers, Oxides and semiconductors, 1997-
  Butler, MacLaren, and Mathon

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

1995-2000

- Predictions of very large TMR for MgO, 2001
  Butler *et al*., Mathon and Umerski

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

2001

The calculated optimistic TMR ratio is in excess of 1000% for an MgO barrier

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

•Experimental confirmation of predictions of high TMR for MTJ’s with MgO barriers 2004, Yuasa

Ohno *et al*.

TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

1995-2000

- CMOS technology, merging spintronics with semiconductors
- Spin injection into semiconductors- 2000 Schmidt et al., Fert & Jaffrès; the Spin transistor-1990 Datta & Das

Resistance mismatch

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Spin transfer

2000-2005

2000-2005

- How charge current produce spin currents which lead to torques acting on background magnetization; back in 1989 JC Slonczewski had the following idea for magnetic tunnel junctions.

- Spin currents produce torques-1996, Berger, Slonczewski

2000-2005

- Experimental confirmation of current driven magnetization reversal (switching)-CIMS-2000,

PHYSICAL REVIEW LETTERS VOLUME **84**, 3149 (2000)

Current-Driven Magnetization Reversal and Spin-Wave Excitations in CoCuCo Pillars

J. A. Katine, F. J. Albert, and R. A. Buhrman
School of Applied and Engineering Physics, Cornell University, Ithaca, New York 14853
E. B. Myers and D. C. Ralph
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853

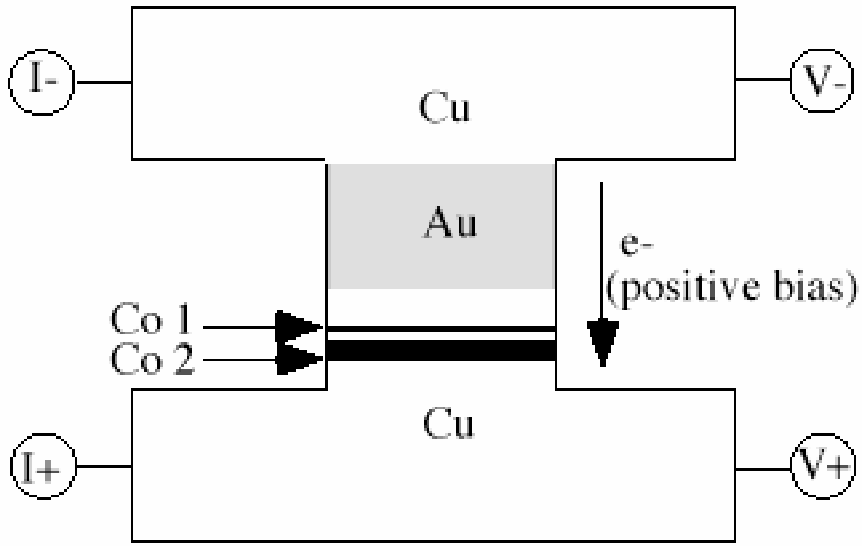


FIG. 1. Schematic of pillar device with Co (dark) layers separated by a 60 Å Cu (light) layer. At positive bias, electrons flow from the thin (1) to the thick (2) Co layer.

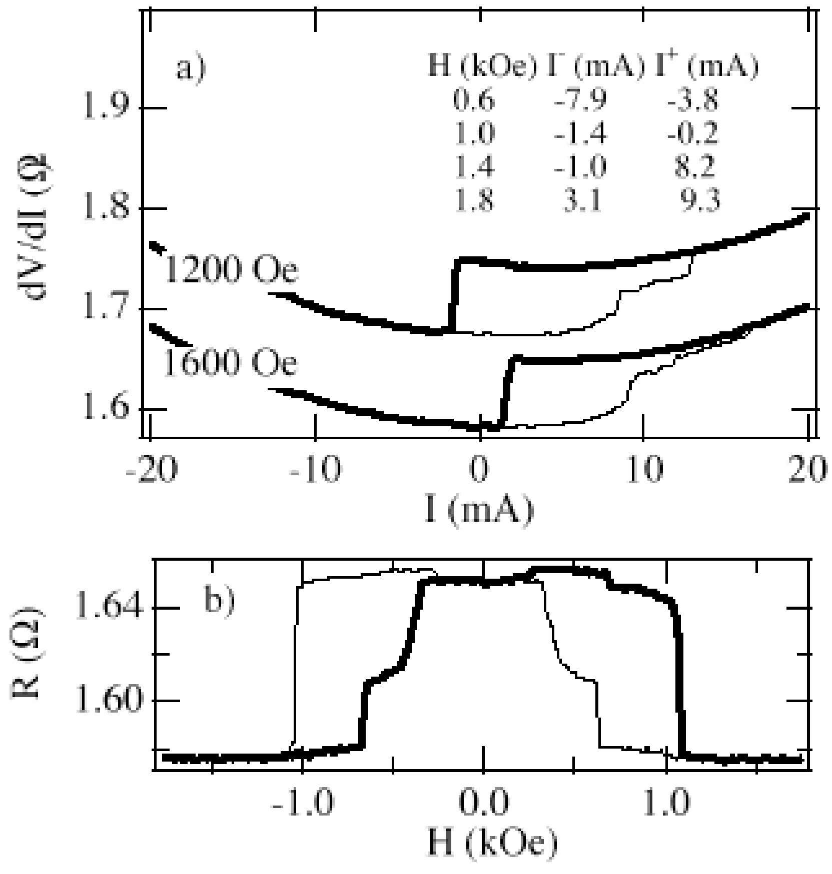
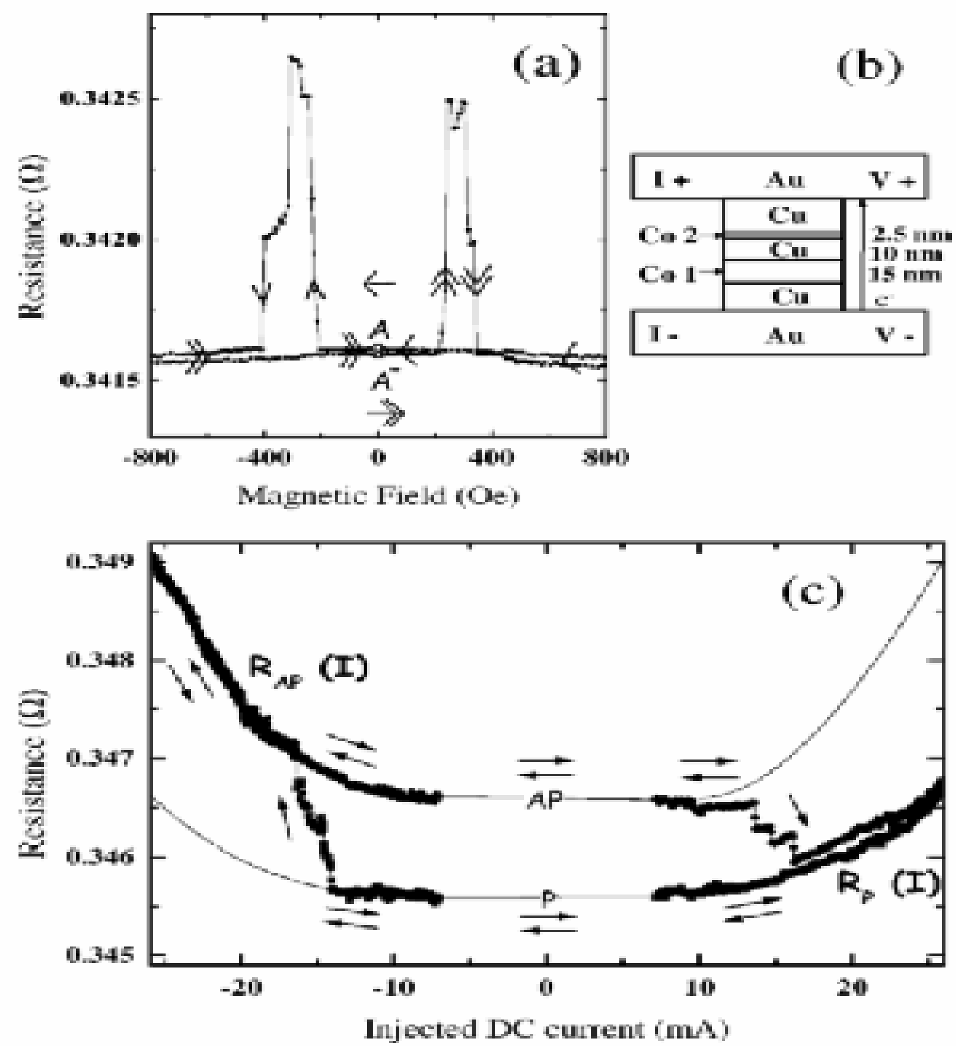


FIG. 2. (a) dV/dI of a pillar device exhibits hysteretic jumps as the current is swept. The current sweeps begin at zero; light and dark lines indicate increasing and decreasing current, respectively. The traces lie on top of one another at high bias, so the 1200 Oe trace has been offset vertically. The inset table lists the critical currents at which the device begins to depart from the fully parallel configuration (I^+) and begins to return to the fully aligned state (I^-). (b) Zero-bias magnetoresistive hysteresis loop for the same sample.

Spin-polarized current induced switching in Co/Cu/Co pillars

J. Grollier, V. Cros, A. Hamzic,[a] J. M. George, H. Jaffrès, and A. Fert
Unité Mixte de Physique CNRS/Thales,[b] 91404 Domaine de Corbeville, Orsay, France
G. Faini
LPN-CNRS, 196 av. H. Ravera, 92225 Bagneux, France
J. Ben Youssef and H. Legall
Laboratoire de Magnétisme de Bretagne-CNRS, 29285 Brest, France

3664 Appl. Phys. Lett., Vol. 78, No. 23, 4 June 2001



How can one rotate a magnetic layer with a spin polarized current?

By spin torques:

Slonczewski-1996

Berger -1996

Waintal et al-2000

Brataas et al-2000

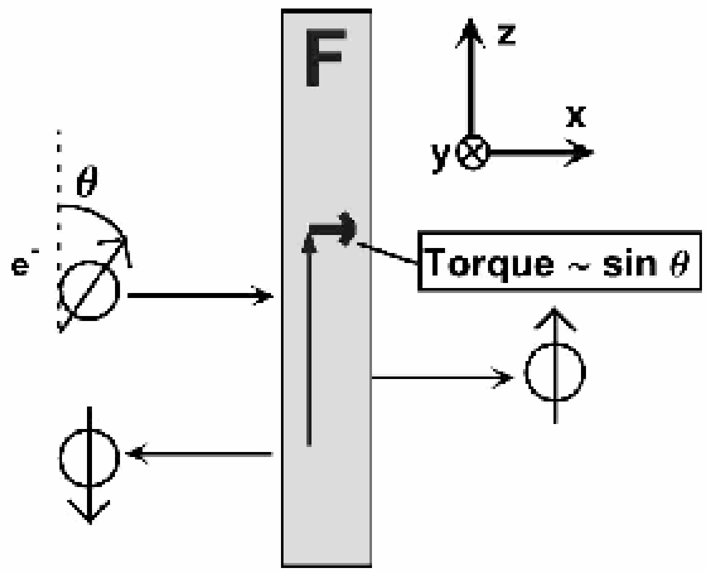


FIG. 1. Schematic of exchange torque generated by spin filtering. Spin-polarized electrons are incident perpendicularly on a thin ideal ferromagnetic layer. Spin filtering removes the component of spin angular momentum perpendicular to the layer moments from the current; this is absorbed by the moments themselves, generating an effective torque on the layer moments.

2000-2005

- Current driven motion of domain walls- 1986-89 Berger exp't. confirmation 2003-, Fert, Ono, Ohno, Rüdiger.

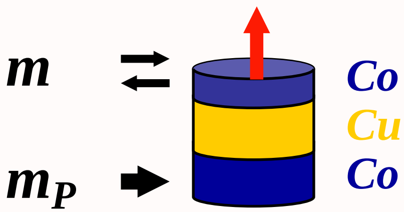
- Spin transfer oscillators STO's, or
  spin transfer driven FMR 2006 - Sankey, Buhrman & Ralph; Boulle, Barnas, Fert

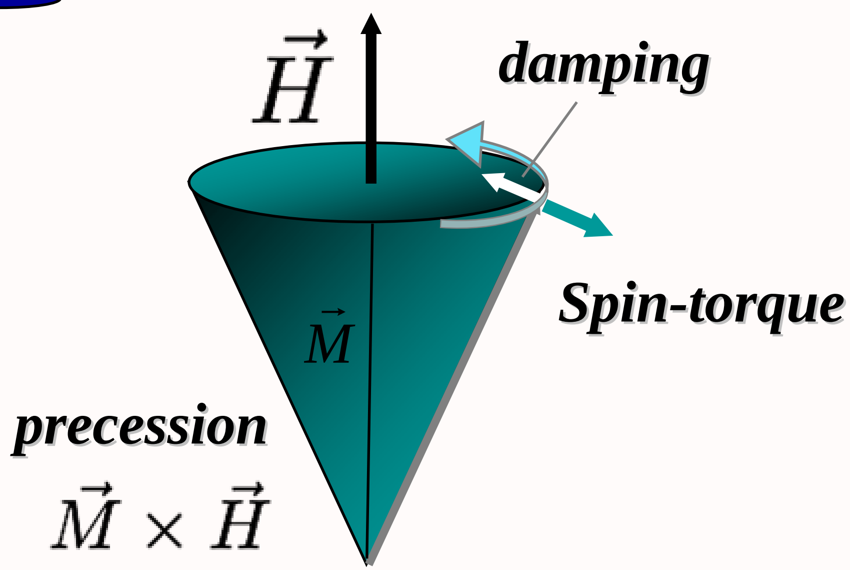
Spin-Transfer Induced Magnetization Reversal

Why is magnetization reversal "slow" with spin-transfer?

- **An initial deviation of the magnetization of the fixed and free layers is necessary.**
- **Usually the reversal occurs over many precession cycles**

$$\frac{d\vec{m}}{dt} = -\gamma \vec{m} \times \vec{H}_{eff} + \alpha \vec{m} \times \frac{d\vec{m}}{dt} + \gamma a_I \vec{m} \times (\vec{m} \times \vec{m}_p)$$





Longitudinal relaxation time:

$$T_1 \sim \frac{1}{\alpha_{eff}} T_p \qquad T_p = \frac{2\pi}{\gamma H}$$

$\gamma = 1.76 \times 10^{11}$/Ts

H=1 T; T_p=36 psec

2000-2005

- Relativistic treatment of torque, the transfer of orbital angular momentum 2006- Weinberger, Györffy

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime?and
TIFF (Uncompressed) dec
are needed to see this p

QuickTime?and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime?and a
TIFF (Uncompressed) decompressor

The present
2006-

2006 -

- Spin Hall effect 1999- , Hirsch, S. Zhang, Sinova

- Semiconductors as barriers; ferromagnetic semi-conductors as electrodes in MTJ's 1968~2000- Kasuya,Wachter, von Molnar, Methfessel, Mattis; 2000- Ohno, Munekata, Dietl, Chiba, Das Sarma, Samarth,Awschalom, MacDonald, Sinova, Wunderlich, Halperin, Brataas, Inoue, Bauer.

- MTJ's with spin filtering barriers, ~2004 - Moodera, Thales group (Barthélemy, Bibes, Gajek,…), Grünberg,

- Multiferroics magnetoelectrics, 2005- Tsymbal, Thales.

2006 -

- Carbon nanotubes ~2000- see review by Roche *et al*. RMP**79**,677 (2007). As applied to spintronics, see Hueso *et al*. Nature **445**, 24 January 2007.

- Graphene, massless Dirac Fermions ~2005- Kim,.. see review by Geim *et al*. in RMP 2007, to appear (con-mat/0709.1163).

- Molecular spintronics ~2000- Ratner, Reed, McEuen, Sanvito


That's all for today folks

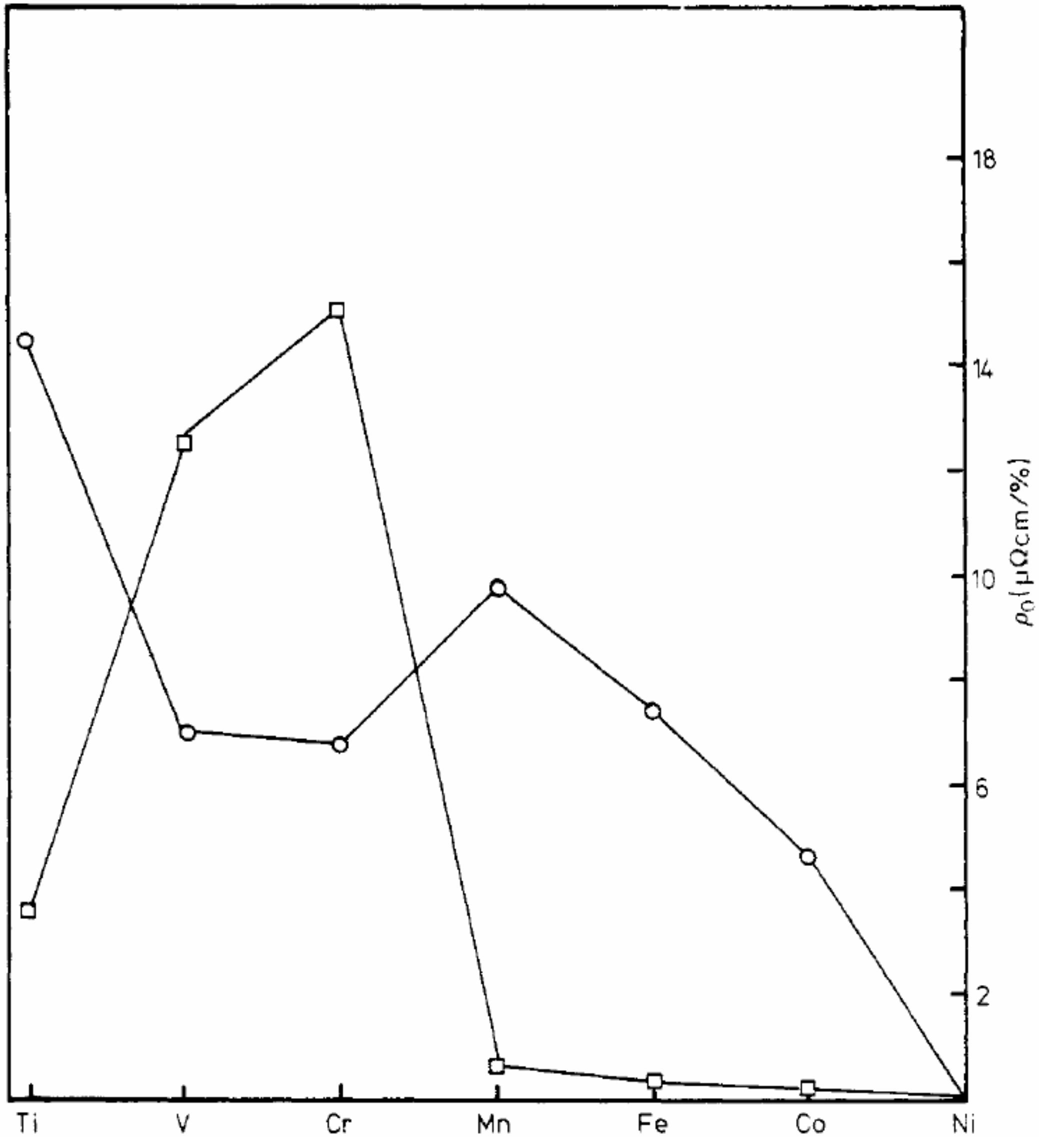


Figure 2. The resistivity of 3d impurities in nickel for each spin direction. □, $\rho_{0\uparrow}$; ○, $\rho_{0\downarrow}$.

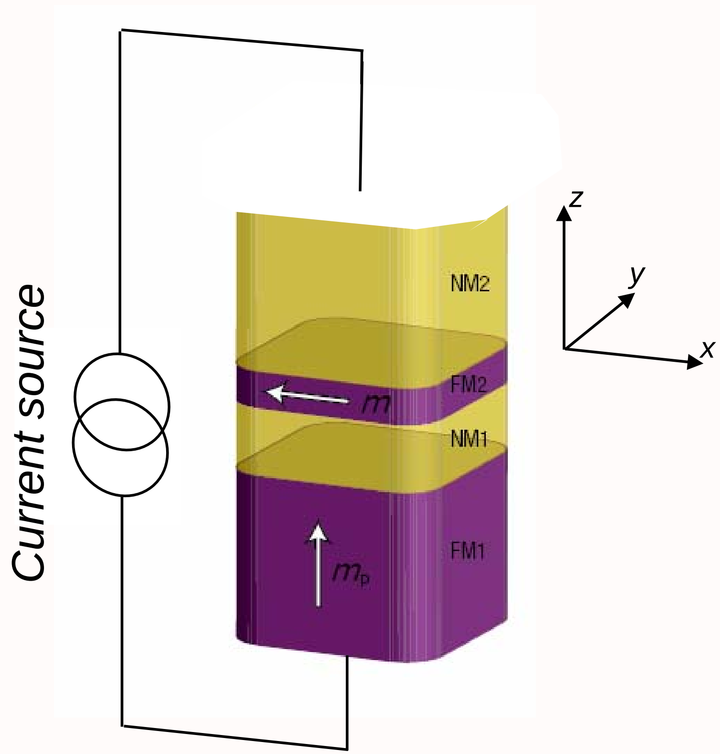
Fert’s interpretation of these results:

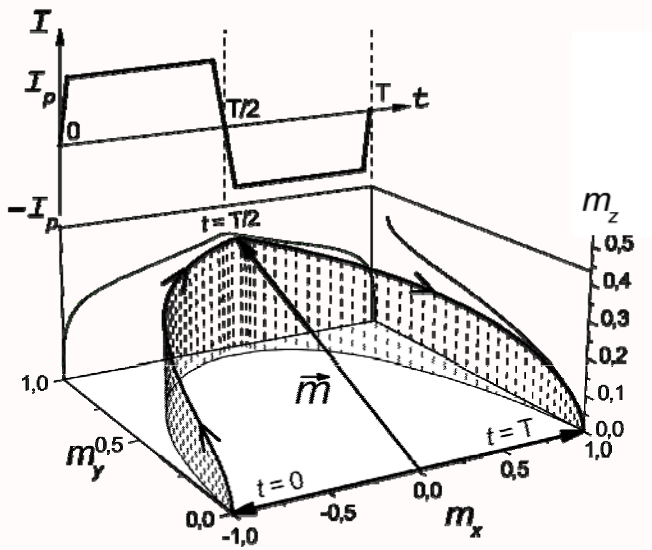
When the difference in the number of 3d electrons between the nickel and the impurity is large (Cr,V,Ti) a spin up d bound state is repelled above the spin up d band. This explains, for instance, that the magnetic moment of Cr, V or Ti impurities is *opposite* to the nickel moment….

For *V* and *Ti,* the spin up resistivity remains rather large, which seems to show that, even for *Ti*, the vbs is not repelled well above the Fermi level,

For *Co, Fe* and *Mn* impurities spin up resistivity is very small. This is due to the presence of only *s* states at the Fermi level for the spin up direction and, in the absence of resonance
effects, to the weakness of s-s scattering.

New concept invented at NYU: Spin-Current Induced Precessional Magnetization Reversal





Minimum time for magnetization reversal

$$\tau \approx 1/(\gamma 4 M)$$

For $\mu_o M$=1 T → $\tau \approx 20\,ps$

NYU has 3 patents and 6 pending appl. and has licensed this IP

ADK et al., APL **84**, 3897 (2004)
ADK, Nature Materials (2007)

Spin transport in magnetic multilayers for condensed matter physicists

Peter M Levy

New York University

Outline

- How charge current creates spin current in ferromagnetic metals

- Giant magnetoresistance

- Transport in collinear multilayers; GMR and TMR

- Noncollinear structures; charge current-resistance and spin current-spin torque. Plane of spin polarization

Metallic multilayers

- Fermi sea and surface
- Scattering in layers, and interfaces between layers
- Diffusive regime for transport; Kubo and Boltzmann
- Out of equilibrium effects; current driven accumulations
- Non-equilibrium Greens functions; Keldysh and Boltzman approaches

Magnetic tunnel junctions

- Conduction controlled by barrier
- Ballistic regime of transport; Landauer formalism
- TMR at zero bias
- Finite bias; inelastic processes, e.g., magnon production
- Non collinear effects; spin torque

Distinction between equilibrium and out-of-equilibruim (transport) effects due to spin currents

In equilibrium: magnetic coupling, e.g., RKKY. Its *amplitude*-strength can be large, i.e., a good fraction of a Bohr magneton, however its *range* is short; several nm.

Out of equilibrium: current driven spin accumulation that has *long range*-can be submicron, but whose *amplitude is tiny*, i.e., one-thousandths of a Bohr magneton. Can couple magnetic regions that are uncoupled in Equilibrium.

Spintronics - control of current through spin of electron
Role of band structure

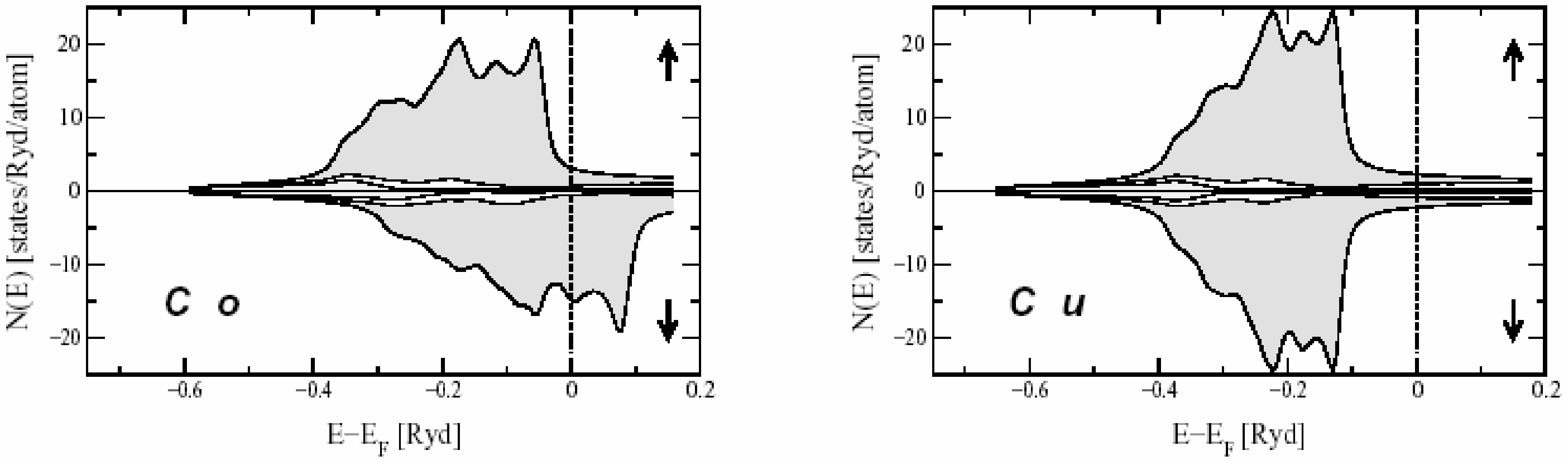


Figure 2.6: Spin-projected densities of states for Co and Cu; the differently shaded grey areas indicate the amount of s, p and d states.

Two quite different manifestations of transport in magnetically layered structures

Charge currents can have resistance dependent on the magnetic configuration-GMR

Spin currents can alter the magnetic configuration through their polarization-spin torque

1st manifestation

The two current model of conduction in ferromagnetic metals

spin. When we exclude spin-flip scattering processes and make the gross yet conventional assumption that all k states for a spin direction scatter at the same rate we arrive at a particularly simple parameterization of conduction in which one assigns a scattering rate or resistivity to each spin channel of conduction (see also Eq. 2.30):

$$\rho^{\uparrow,\downarrow} = \rho^{M,m} \equiv a \pm b, \tag{2.77}$$

where the superscripts M and m, refer to electrons with spin parallel (Majority) and opposite (minority) to the magnetization. From Eq. (2.65) and the realization that for a homogeneous sample the effective fields are independent of spin, the total current in the two spin channels is,

$$j = j^{\uparrow} + j^{\downarrow} = (\sigma^{\uparrow} + \sigma^{\downarrow})E, \tag{2.78}$$

so that the resistivity is

$$\rho = \frac{1}{\sigma^{\uparrow} + \sigma^{\downarrow}} = \frac{\rho^{\uparrow}\rho^{\downarrow}}{\rho^{\uparrow +}\rho^{\downarrow}} \\ = \frac{a^2 - b^2}{2a}. \tag{2.79}$$

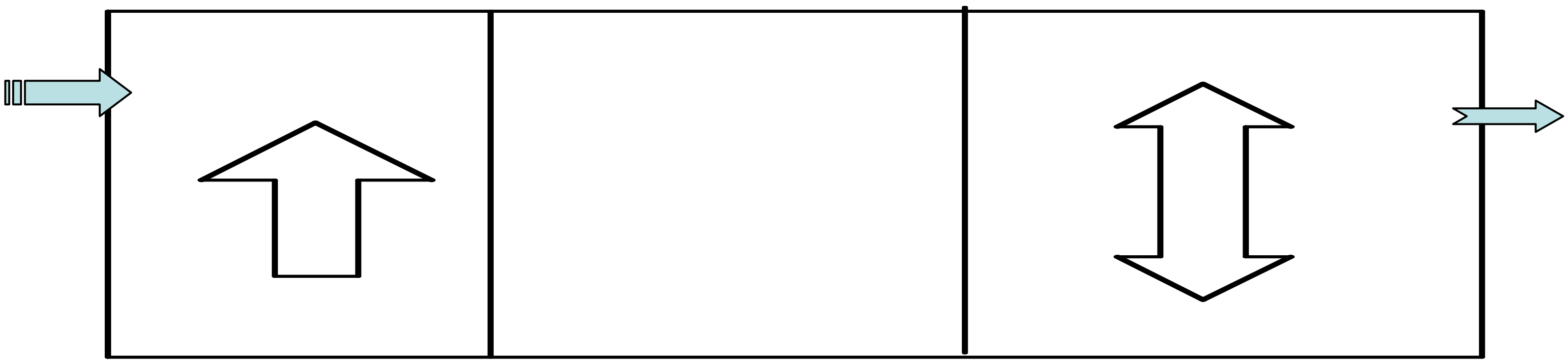
It is obvious that the resistance in ferromagnetic metals is less than in materials with comparable scattering rates, i.e., the average resistivity for each channel is a, so that for two channels conducting in parallel we find $\rho = 1/2a$, while from Eq. (2.79) we find $\rho < \frac{1}{2a}$. The channel with the lower resistivity conducts more of (shunts) the current, and creates the effect of a *short circuit.*

Electrical resistance will be traced to the loss of momentum information of electrons; the Drude formula

$$\varrho = \frac{m}{ne^2\tau} \tag{2.3}$$

1988 Giant magnetoresistance

Albert Fert & Peter Grünberg



Parallel configuration Antiparallel configuration

Two current model in magnetic multilayers

$$\rho^{\uparrow} = \frac{1}{4}\left[2\rho^{M} + 2\rho^{n}\right],$$

$$\rho^{\downarrow} = \frac{1}{4}\left[2\rho^{m} + 2\rho^{n}\right],$$

$$\rho^{\uparrow} = \rho^{\downarrow} = \frac{1}{4}\left[\rho^{M} + \rho^{m} + 2\rho^{n}\right],$$

$$\rho_P = \frac{1}{4}\left\{(a+c) - \frac{b^2}{(a+c)}\right\},$$

$$\rho_{AP} = \frac{1}{4}(a+c).$$

Data on GMR

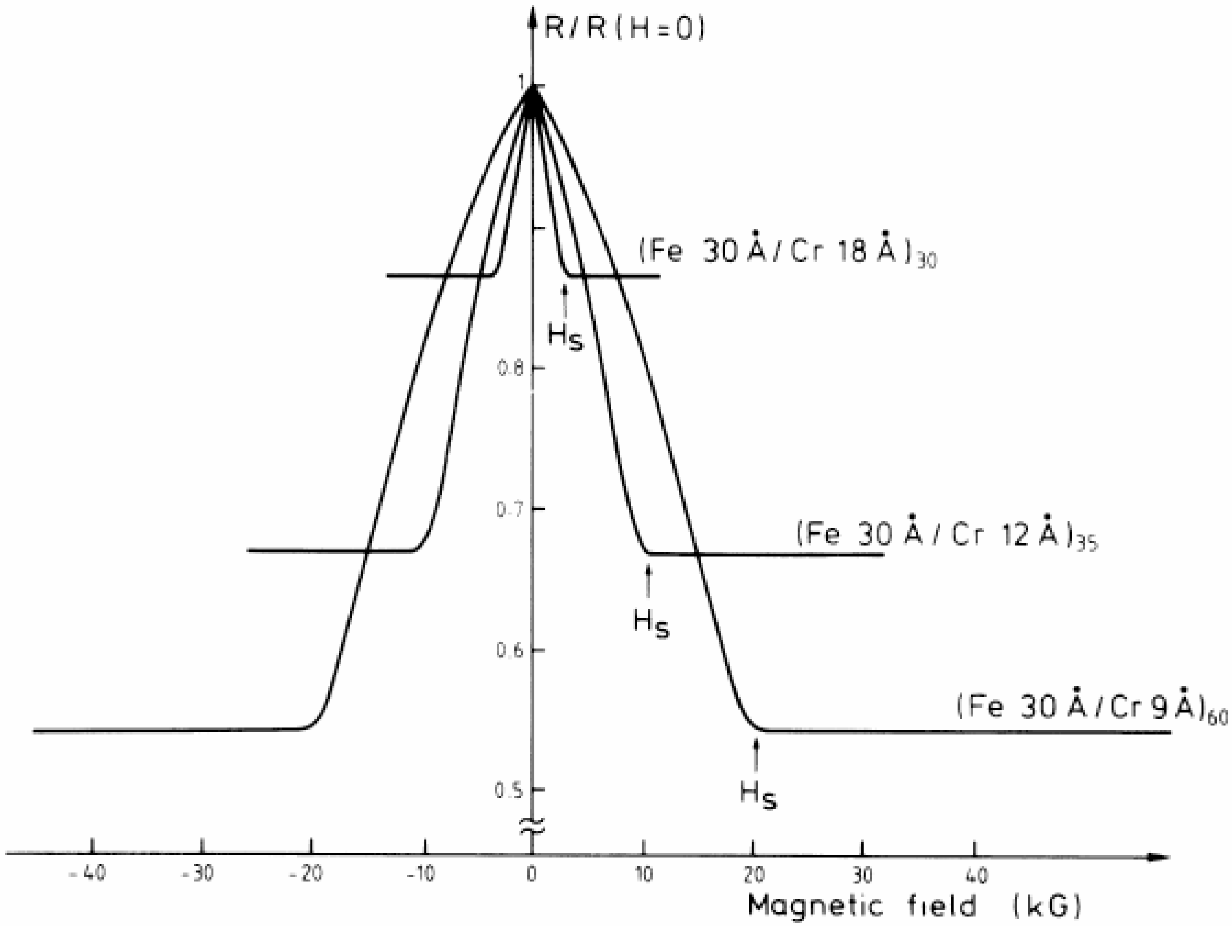
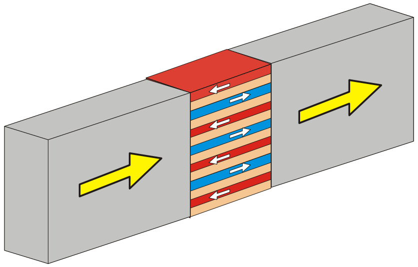


FIG. 3 Magnetoresistance of three Fe/Cr superlattices at 4.2 K. The current and the applied field are along the same [110] axis in the plane of the layers.

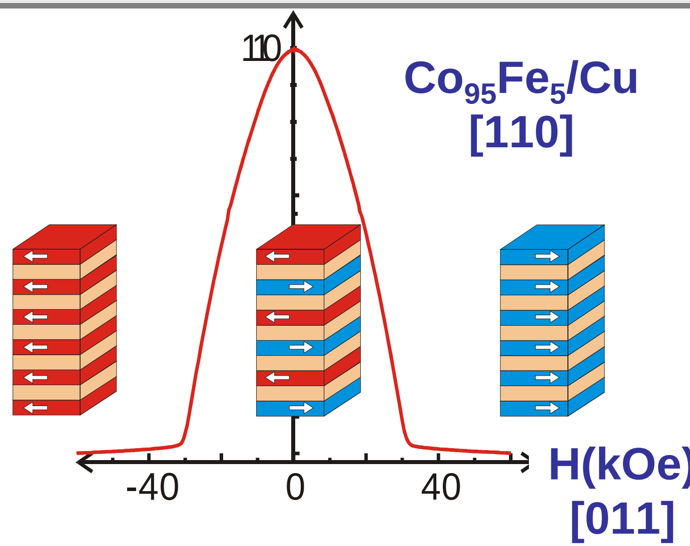
M.N. Baibich *et al.*, Phys. Rev. Lett. **61**, 2472 (1988).

GMR in Multilayers and Spin-Valves

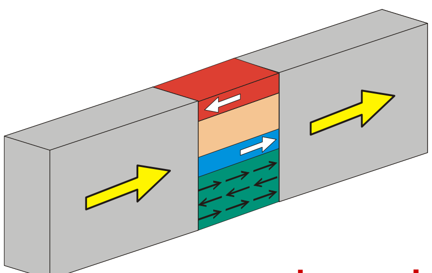


multi-layer

ΔR/R~110% at RT
Field ~10,000 Oe

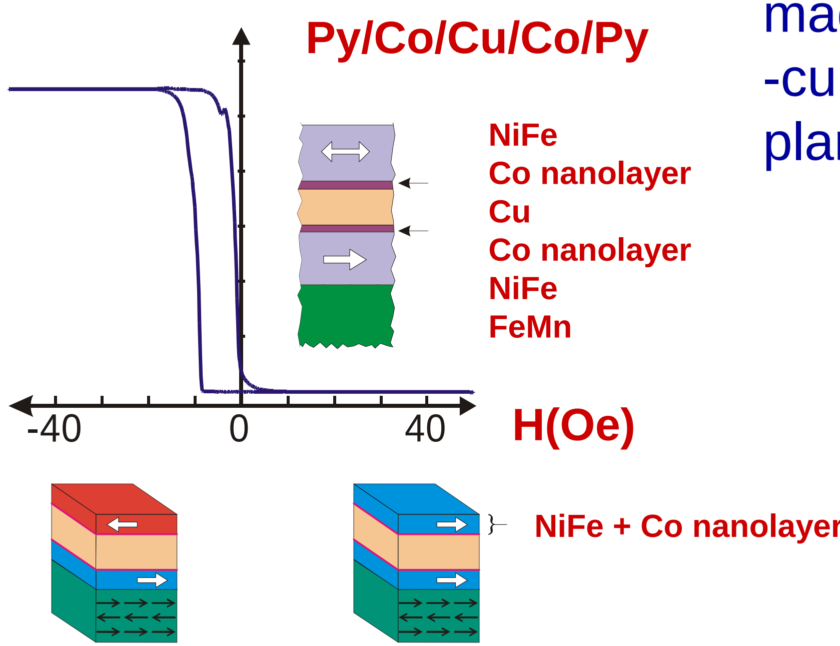


GMR
-metallic spacer between magnetic layers
-current flows in-plane of layers



spin-valve

ΔR/R~8-17% at RT
Field ~1 Oe



S.S.P. Parkin

QuickTime?and a TIFF (Uncompressed) decompressor are needed to see this picture.

From IBM website; http://www.research.ibm.com/research/gmr.html

2nd manifestation

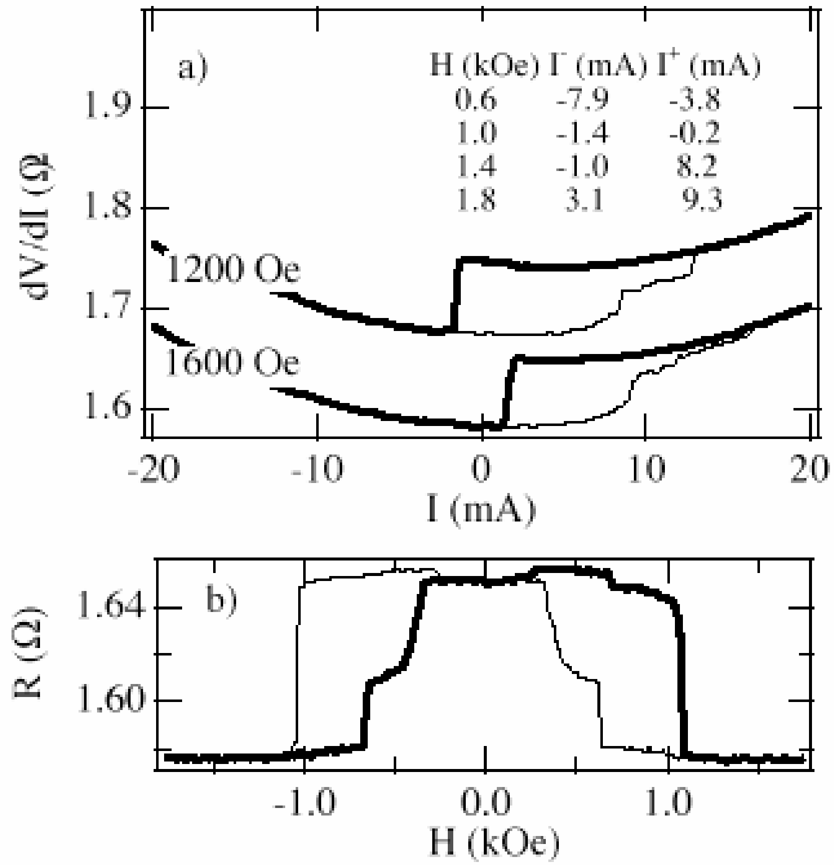
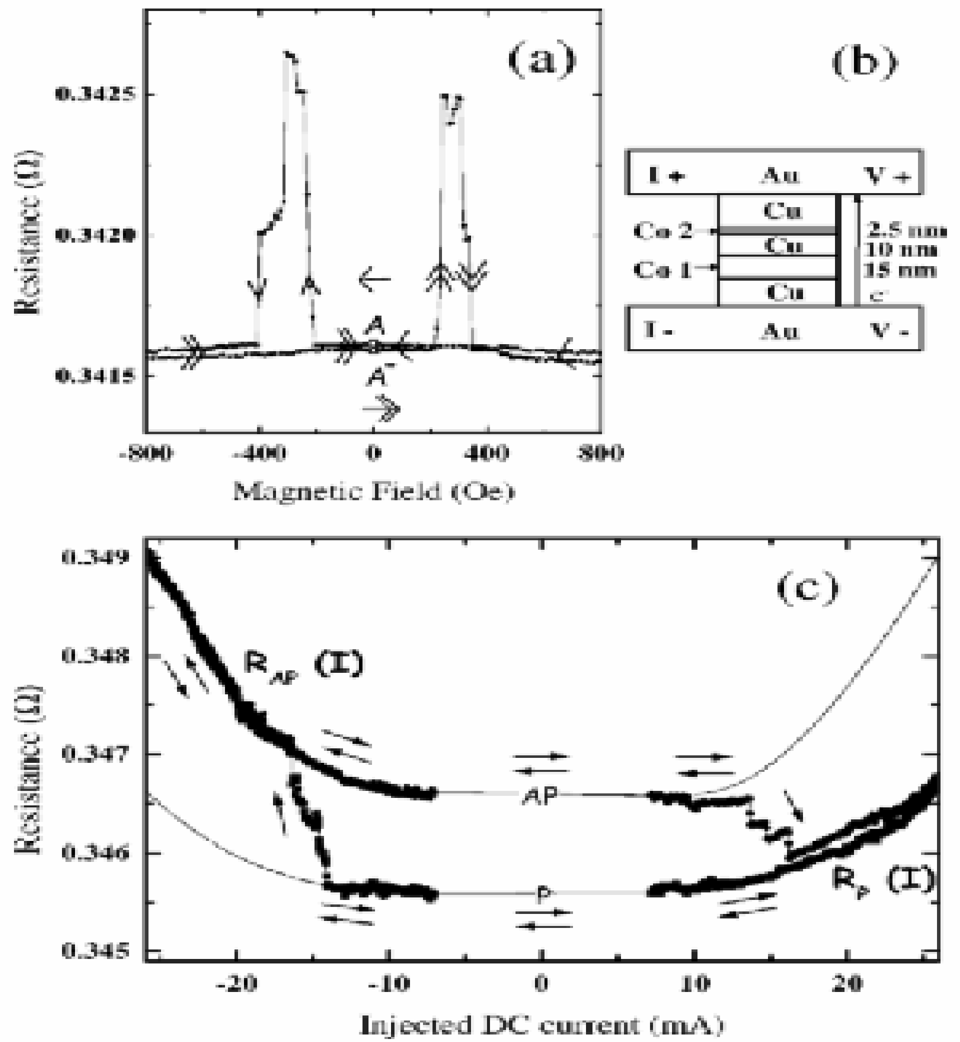


FIG. 2. (a) dV/dI of a pillar device exhibits hysteretic jumps as the current is swept. The current sweeps begin at zero; light and dark lines indicate increasing and decreasing current, respectively. The traces lie on top of one another at high bias, so the 1200 Oe trace has been offset vertically. The inset table lists the critical currents at which the device begins to depart from the fully parallel configuration (I^+) and begins to return to the fully aligned state (I^-). (b) Zero-bias magnetoresistive hysteresis loop for the same sample.

Spin-polarized current induced switching in Co/Cu/Co pillars

J. Grollier, V. Cros, A. Hamzic,[a)] J. M. George, H. Jaffrès, and A. Fert
Unité Mixte de Physique CNRS/Thales,[b)] 91404 Domaine de Corbeville, Orsay, France
G. Faini
LPN-CNRS, 196 av. H. Ravera, 92225 Bagneux, France
J. Ben Youssef and H. Legall
Laboratoire de Magnétisme de Bretagne-CNRS, 29285 Brest, France

3664 Appl. Phys. Lett., Vol. 78, No. 23, 4 June 2001



How can one rotate a magnetic layer with a spin polarized current?

By spin torques:

Slonczewski-1996

Berger -1996

Waintal et al-2000

Brataas et al-2000

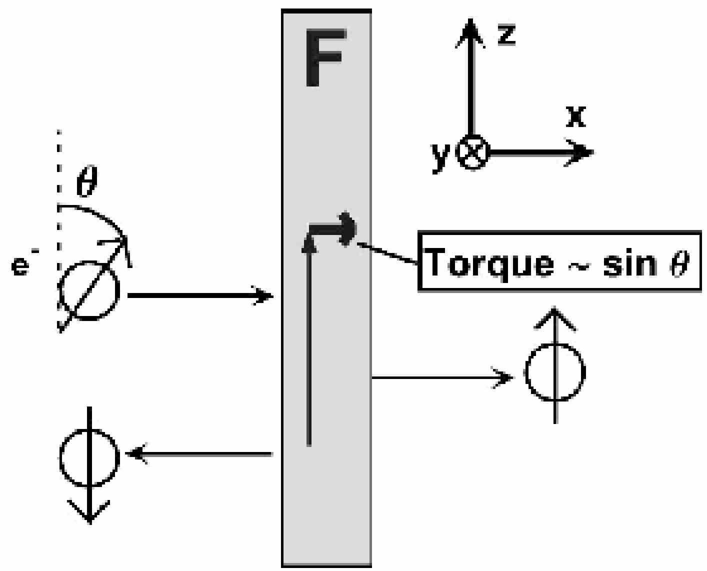


FIG. 1. Schematic of exchange torque generated by spin filtering. Spin-polarized electrons are incident perpendicularly on a thin ideal ferromagnetic layer. Spin filtering removes the component of spin angular momentum perpendicular to the layer moments from the current; this is absorbed by the moments themselves, generating an effective torque on the layer moments.

By current induced interlayer coupling:

Heide- 2001

To discuss transport two calculations are necessary:

- Electronic structure, and
- Transport equations; out of equilibrium collective electron phenomena.

Structures

- Metallic multilayers
- Magnetic tunnel junctions
  - Insulating barriers
  - Semiconducting barriers
  - Half-metallic electrodes
  - Semiconducting electrodes

} different length scales

Lexicon of transport parameters

Spin independent transport

$$\varepsilon_F = \text{ Fermi energy}$$

$$v_F = \text{Fermi velocity} = \frac{1}{\hbar}\frac{\partial \varepsilon}{\partial k}$$

$$k_F = \text{Fermi momentum}$$

$$\tau_{mfp} = \text{ Mean time between collisions}$$

$$\lambda_{mfp} = \text{Distance travelled between collisions}$$

$$\Rightarrow \overline{G}(r-r',\varepsilon_F) \propto e^{i(k_F + i/\lambda)|r-r'|}$$

$$\approx v_F \tau_{mfp}$$

Spin dependent transport parameters

$$\tau_s = \text{Spin dependent relaxation time } s = \uparrow, \downarrow / M, m$$

$$\tau_{sf} = \text{ Time between spin flips}$$

$$\lambda_{sdl} \cong \sqrt{\lambda_{sf} \lambda_{mfp}} = \text{Spin diffusion length}$$

$$d_J = {hv_F}/{J} = \text{Spin coherence length}$$

due to temporal precession; J = echange constant

$$\lambda_{tr} = \text{Transverse spin coherence length}$$

$$\cong \lambda_J \cong \sqrt{d_J \lambda_{mfp}} = \text{transverse spin diffusion length}$$

$$l_c = 1 / \left| k_{F\uparrow} - k_{F\downarrow} \right| = \text{ Transverse spin coherence length}$$

due to spatial precession.

Comparisons

• For conduction in metallic multilayers **electrons sample all regions** more or less **equally** and transport is **diffusive** in to dates multilayers.

Kubo formalism treats transport diffusively via random impurity averages (CPA), yet it can restore correlations through vertex corrections; Kubo is my preference for all metal systems.

• In tunnel junctions electrons are inordinately **sensitive to electrode/barrier interface.**

Caroli formalism is an easy way to discriminate between different regions of the system. It is, therefore, able to describe transport i terms of the properties of the interface region. Caroli is my choic for tunnel junctions

Diffusive transport

Collisions assure local equilibrium of current; however $a << \lambda_{mfp} << L,$ where a is lattice constant. Also, $\lambda_{mfp} <<$ phase coherence length of wavefunctions.

In this regime one can usually describe transport by semi-classical Boltzmann equation. This is an equation of motion for an electron distribution function, $f(r,k,t)$.

$$\partial f/\partial t + \mathbf{v}\cdot\nabla f - eE\cdot\mathbf{v}\delta(\varepsilon-\varepsilon_F)$$
$$= -1/\tau\{f - \langle f\rangle\}$$

Charge and spin current and accumulation

$$\hat{n}(r) \equiv \int_{\varepsilon_F} \frac{d\hat{k}}{4\pi} \hat{f}(\hat{k}, r) \qquad \hat{j}(r) \equiv \int_{\varepsilon_F} \frac{d\hat{k}}{4\pi} v(\hat{k}) \hat{f}(\hat{k}, r)$$

$$\hat{f}(\hat{k}, r) = f_0 \mathbf{1} + \vec{f} \cdot \sigma$$

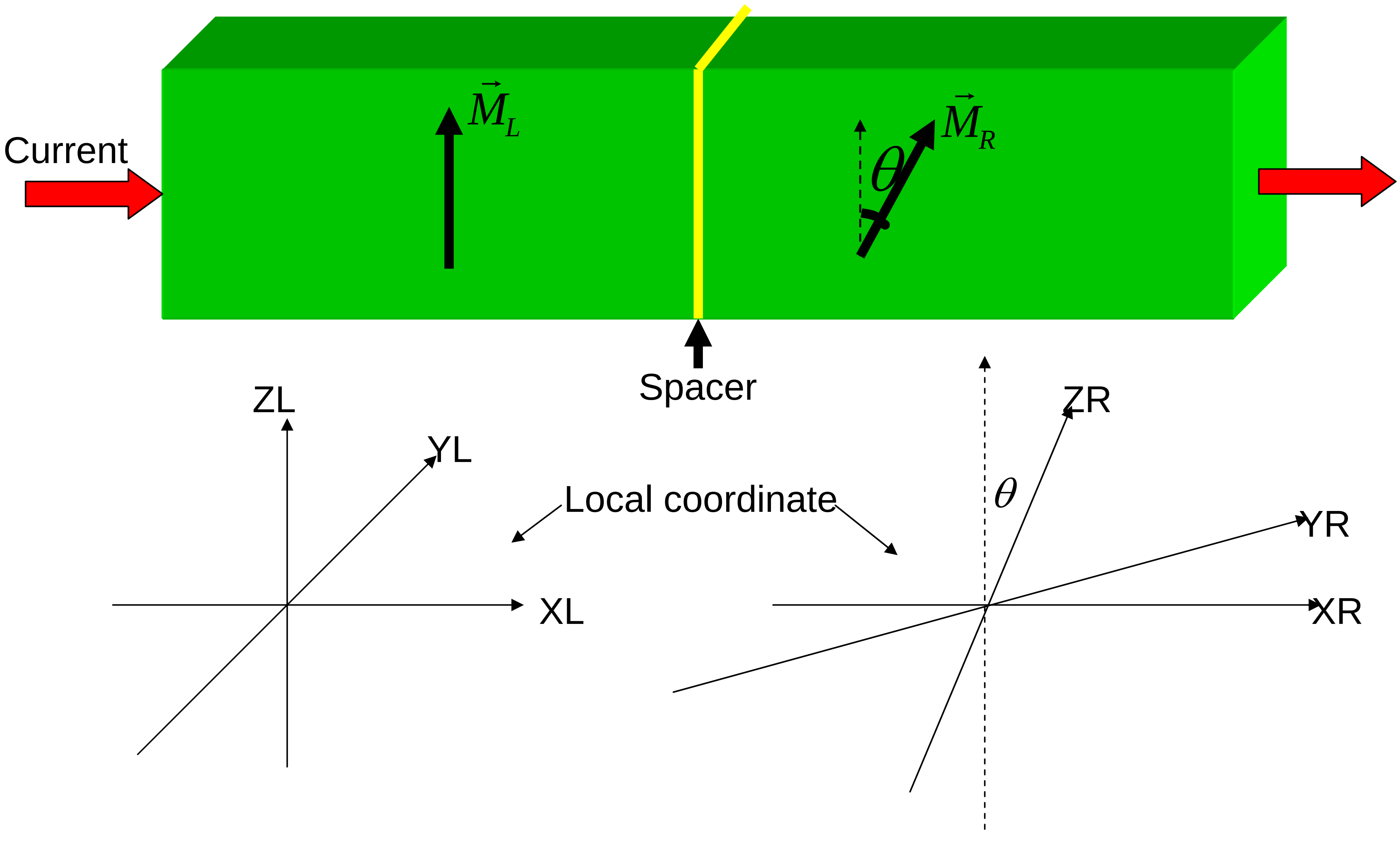
Charge accumulation and current :

$$q(r) = \int_{\varepsilon_F} \frac{d\hat{k}}{4\pi} f_0(\hat{k}, r) \qquad j_e(r) = \int_{\varepsilon_F} \frac{d\hat{k}}{4\pi} v(\hat{k}) f_0(\hat{k}, r).$$

Spin accumulation and current :

$$\vec{m}(r) = \int_{\varepsilon_F} \frac{d\hat{k}}{4\pi} \vec{f}(\hat{k}, r) \qquad \vec{j}_m(r) = \int_{\varepsilon_F} \frac{d\hat{k}}{4\pi} v(\hat{k}) \vec{f}(\hat{k}, r).$$

Two semi-infinite magnetic layers of same material



For multilayered structures one usually finds the current by solving an equation of motion for it within each layer and then connect solutions across the interfaces between layers by using matching conditions that come from the scattering coefficients.

For electron transport across noncollinear magnetic layers two problems arise:

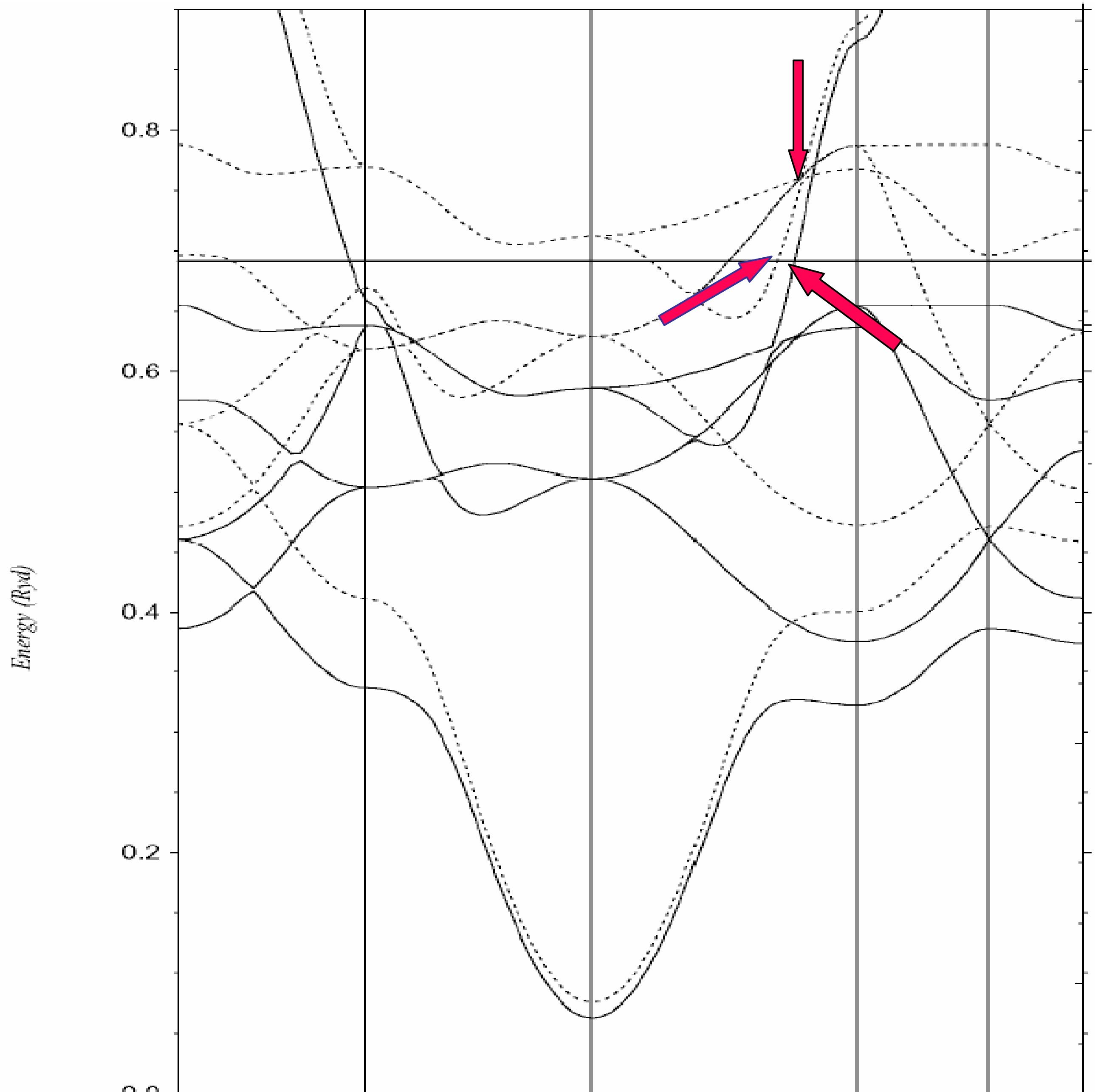
- What is the correct form of the out of equilibrium distribution function, and the equation of motion it satisfies?

- What are the scattering coefficients that one should use to connect solutions in adjacent layers?

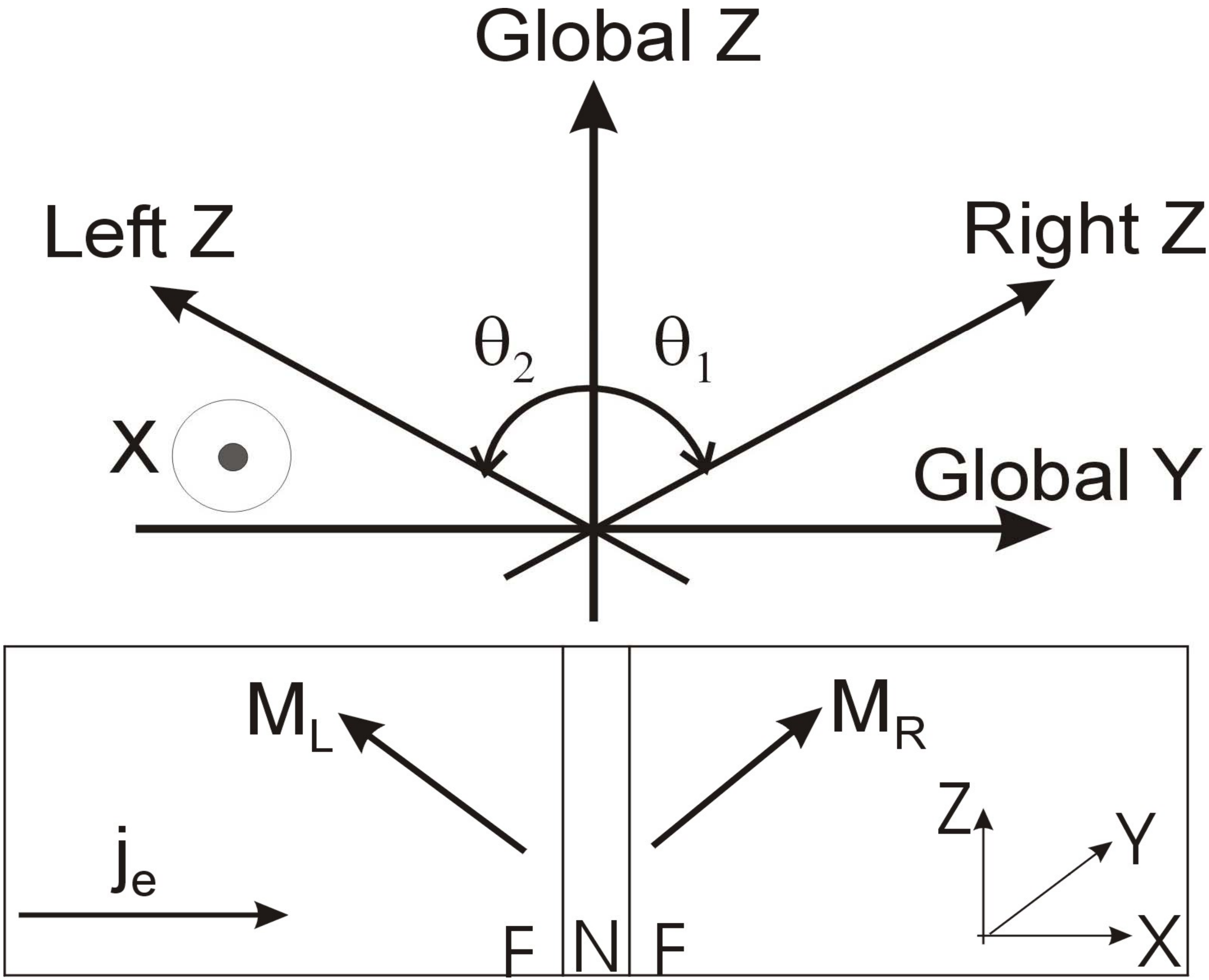
Spin currents in noncollinear magnetic multilayers
A comparison

Property	Conventional	Our method
Transverse spin current	Discontinuous at interface	Continuous across interface
Transverse accumulation	Not self-consistent	Determined self consistently
Origin of spin torque	Discontinuity in spin current	Transverse spin accumulation
Injection of transverse spin current	No way to inject transverse spin distribution into ferro-magnetic layer	Current induced spin flip at interface
Mode of propagation	Transverse current carried on 2 sheets of Fermi surface	Carried on one sheet
Steady state	Constant loss of transverse component of spin current	Achieved through transverse spin accumulation

Band structure of Co



Steady state calculation
Local axis coordinates



Methodology: Boltzmann equation using the layer-by-layer approach

Boltzmann equation for spin currents in ferromagnetic metals. See Jianwei Zhang *et al.*, PRL **93**, 256602 (2004).

$$\hat{f}(\mathbf{k},x) = \hat{f}^0(\mathbf{k}) + \left(-\frac{\partial \hat{f}^0}{\partial \hat{\epsilon}}\right)[f(\mathbf{k},x)\hat{1} + \mathbf{g}(\mathbf{k},x)\cdot\boldsymbol{\sigma}],$$

$$\frac{\partial \hat{f}}{\partial t} + \hat{v}_x \frac{\partial \hat{f}}{\partial x} - \frac{eE}{\hbar}\frac{\partial \hat{f}^0}{\partial k_x} + \frac{i}{\hbar}[\hat{\varepsilon}, \hat{f}] = -\left(\frac{\partial \hat{f}}{\partial t}\right)_{\text{collision}},$$

Boltzmann equation

$$\hat{F} = \frac{1}{2}\left(f_0 * \hat{I} + \vec{f} \cdot \vec{\sigma}\right) \qquad \hat{\tau}^{-1} = \frac{1}{2}\left(\tau_0^{-1} * \hat{I} + \vec{\tau}^{-1} \cdot \vec{\sigma}\right) \qquad <\hat{F}(x)> = \frac{1}{4\pi}\int \hat{F}(v_x, x)\, d\Omega$$

$$v_x \frac{\partial \hat{F}}{\partial x} - eEv_x \frac{\partial \hat{F}^0}{\partial \varepsilon} + \left(\frac{J}{\hbar}\right)\vec{M} \times \vec{f} = -\frac{1}{2}[\hat{\tau}^{-1}(\hat{F} - <\hat{F}>) + (\hat{F} - <\hat{F}>)\hat{\tau}^{-1}] - \frac{1}{\tau_{sf}}[2 < \hat{F} > - \hat{I} * Tr < \hat{F} >]$$

diffusion — Field — Exchange — Non spin flip scattering — spin flip scattering

Solution of Boltzmann equation across interface between *noncollinear* magnetic layers.

At interface between layers

$$\hat{F}_L(0, v_x) = R^{-1}(\theta)\hat{F}_R(0, v_x)R(\theta)$$

Spin current

$$\vec{J}(x) = \frac{1}{4\pi}\int v_x * \vec{f}(v_x, x)\, d\Omega$$

$$\partial_t f_{\uparrow,\downarrow} + (v_x \pm u_x)\frac{\partial f_{\uparrow,\downarrow}}{\partial x} + eE(v_x \pm u_x) = -\frac{f_{\uparrow,\downarrow} - < f_{\uparrow,\downarrow} >}{\tau} - \frac{f_{\uparrow,\downarrow} - < f_{\downarrow,\uparrow} >}{\tau_{sf}}$$

$$\partial_t \mathbf{g}^{\pm} + v_x \frac{\partial \mathbf{g}^{\pm}}{\partial x} \mp i\left(\frac{J_k}{\hbar}\right)\mathbf{g}^{\pm} = -\frac{\mathbf{g}^{\pm} - < \mathbf{g}^{\pm} >}{\tau} - 2\frac{< \mathbf{g}^{\pm} >}{\tau_{sf}}$$

$$\hat{\upsilon}_x = \frac{1}{\hbar}\frac{\partial \hat{\varepsilon}}{\partial k_x} = \frac{1}{\hbar}\left(\frac{\partial \varepsilon}{\partial k_x} + \frac{1}{2}\frac{\partial J}{\partial k_x}\mathbf{M}\cdot\boldsymbol{\sigma}\right)$$
$$\equiv \upsilon_x \hat{1} + u_x \mathbf{M}\cdot\boldsymbol{\sigma}.$$

$$2a_{\pm} = \delta(\boldsymbol{\epsilon}_k + J_k/2 - \boldsymbol{\epsilon}_F) \pm \delta(\boldsymbol{\epsilon}_k - J_k/2 - \boldsymbol{\epsilon}_F)$$

The critical new ingredient in this derivation is that associated with each band crossing there is a spinor distribution function that describes the out of equilibrium electron distribution.

From the viewpoint of the band structure (in equilibrium) this is counterintuitive inasmuch as the states in each band are pure spin states.

Charge current

$$j_e = -e\int d\mathbf{k}\{(a_+v_x + a_-u_x)f + (a_+u_x + a_-v_x)\mathbf{g}\cdot\mathbf{M}\},$$

Circuit theory

$$j_{il}^{(c)} = \left(G_i^{\uparrow} + G_i^{\downarrow}\right)\Delta V_i^{(c)} + \left(G_i^{\uparrow} - G_i^{\downarrow}\right)\Delta\mathbf{V}_i^{(s)}\cdot\mathbf{m}_i$$

Spin current

$$\mathbf{j}_s = -e\int d\mathbf{k}(a_+u_x + a_-v_x)f\mathbf{M} + (a_+v_x + a_-u_x)\mathbf{g},$$

Parallel to magnetization

Due to accumulation

Circuit theory

$$j_{il}^{(s\|)} = \left(G_i^{\uparrow} - G_i^{\downarrow}\right)\Delta V_i^{(c)} + \left(G_i^{\uparrow} + G_i^{\downarrow}\right)\Delta\mathbf{V}_i^{(s)}\cdot\mathbf{m}_i$$

$$\mathbf{j}_{il}^{(s\perp)} = -2G_i^{\uparrow\downarrow}\mathbf{m}_i\times\mathbf{V}_{il}^{(s)}\times\mathbf{m}_i\,,$$

Definition of transverse spin current, in the steady state

$$
\begin{aligned}
j^{\pm}(x>0) &= \int_{FS} d\mathbf{k} v_x g_R^{\pm}(k, x>0) \\
&= \int_{FS} d\mathbf{k} c_{\pm}^{>}(\mathbf{k}) v_x \exp(\pm i \frac{J_k}{\hbar v_x} x) \\
&\sim \int_{FS} d\mathbf{k} v_x \exp(\pm i \frac{J_k}{\hbar v_x} x),
\end{aligned}
$$

•In a statistical density matrix, e.g., the Boltzmann distribution function, there are diagonal matrix elements which represent populations, and the off diagonal which are coherences between states.

•For noncollinear multilayers one must be mindful of coherences.

•In equilibrium magnetic layers are not magnetically coupled; in the presence of a spin current across a normal spacer the scattering at the opposite interfaces of the spacer interact with one another, e.g., see Valet and Fert PRB **48**, 7099 (93).

•*CISP's* is our way of introducing in a steady state calculation transients that admix excited k states into the ground state so as to arrive at the correct steady state.

Within a ferromagnetic layer an electric field only gives rise to a longitudinal spin current; transverse spin currents only arise from injecting a longitudinal spin current created in an adjacent layer whose magnetization is at an angle to the layer under consideration.

While the equations of motion allow a transverse current to propagate over a distance related to their scattering rate, the method of creating them by injection, as described above, has currents with different momentum directions relative to the electric field interfere as I will presently show.

Therefore while the natural decay length for a transverse current is of the order of 5-10 nm, the destructive interference pattern has them disappear in about 3 nm in Co for example.

Solution for multilayer is to find distribution function in each layer by using Boltzmann equation. To determine unknown constants one has to match functions across layers by using the transmission and reflection coefficients.

For example, for transverse distribution function

$$g_R^{\pm}(k, x > 0) = c_{\pm}^{>} \exp(\pm i \frac{J_k}{\hbar v_x} x);$$

$$c_{+}^{>} = T_{MMMM'} f_{MM}(0^-) + R_{MMMM'} f_{MM}(0^+)$$

$$c_{-}^{>} = T_{MMM'M} f_{MM}(0^-) + R_{MMM'M} f_{MM}(0^+)$$

Connection formulae across interfaces
see P.M. Levy and Jianwei Zhang, PRB **70**, 132406 (2004)

To relate distribution functions $f_{ss'}(k,r)$ across an interface one resorts to the transfer matrix[4]

$$f^{>}_{ss'}(k,0^{+}) = \sum_{mm'\hat{k}'} T_{mm'ss'} f^{>}_{mm'}(\hat{k}',\varepsilon_k,0^{-}) + \sum_{mm'\hat{k}''} R_{mm'ss'} f^{<}_{mm'}(\hat{k}'',\varepsilon_k,0^{+}), \quad (1)$$

where the transmission and reflection coefficients (probabilities) are

$$T_{mm'\Rightarrow ss'} = T_{mm'ss'} = t_{ms} * t^{*}_{m's'},$$

$$R_{mm'\Rightarrow ss'} = R_{mm'ss'} = r_{ms} * r^{*}_{m's'}, \quad (2)$$

and $t_{ms}(k',k,\varepsilon_k)$ and $r_{ms}(k'',k,\varepsilon_k)$ are the transmission and reflection amplitudes for elastic scattering. The superscripts

At the interface between normal and ferromagnetic layers (N/F) the scattering potential, while spin dependent, is diagonal in spin space; i.e., there is a unique spin direction, and there are no elements $m \neq s$ when the system is in *equilibrium* and $\langle c^{\dagger}_{k_1 m} c_{k_3 s} \rangle \sim n_s^{int} \delta_{sm}$. It also follows that

$$t_{ms}^{eq} = t_s \delta_{sm}.$$

This leads to the “mixing conductance in the conventional view.

$$T_{mm \Rightarrow ss'} \sim [t_s(k_M) t^{*}_{s'}(k_m)]\{i/2 \sin \theta \delta_{s',-s}\}, \qquad (15)$$

For this path of conduction the transverse spin density that appears on the nonmagnetic side of an interface with a unique momentum can only be transferred to the other side on different sheets of the spin split Fermi surface of the ferromagnetic layer.

As these sheets cross the Fermi surface with different momenta, this invariably dooms the transverse spin current to die within 1 nm of the interface.

To inject a transverse spin current in a ferromagnet that is carried by one sheet, and therefore at one momentum, it is necessary to consider the lowering of the symmetry at the N/F due to the spin accumulation induced by the spin current coming from another magnetic layer that is at an angle to the layer in which we are injecting the transverse spin current.

In other words we must consider the out of equilibrium correction to interface scattering, i.e., the current induced changes in the scattering potential.

$$
\begin{aligned}
f^{>}_{ss'}(\hat{k},\varepsilon_F,0^+) &= \sum_{k,m}\{t^{eq}_{ms}\times\delta t^{*}_{ms'}+\delta t_{ms}\times t^{eq}_{ms'}{}^{*}\}f^0(\varepsilon_k,0^-)\\
&= \delta n^{int}_z \sin\,\theta\{\mathrm{Re}\,\overline{t_sV^*_s}[\sigma_{y_r}]_{ss'}+\mathrm{Im}\,\overline{t_sV^*_s}[\sigma_{x_r}]_{ss'}\},
\end{aligned}
\tag{11}
$$

where

$$
\overline{t_sV^*_s}=\int d\varepsilon_k\, f^0(\varepsilon_k)g_n(\varepsilon_k)\langle\langle t_s(k_2,k_n;\varepsilon_k)V^*_s(k_sk_n;\varepsilon_k)\rangle\rangle,
\tag{12}
$$

$$
\begin{aligned}
f^{>}_{ss'}(\hat{k},\varepsilon_F,0^+) &= \sum_m T_{mm\Rightarrow ss}(\varepsilon_F)\delta_{ss'}\delta f_m(\hat{k}')+\sum_m\frac{1}{2}\alpha\sin\,\theta\\
&\quad\times[\sigma_{z_l}]_{mm}\{\mathrm{Re}\,\overline{t_sV^*_s}[\sigma_{y_r}]_{ss'}\\
&\quad+\mathrm{Im}\,\overline{t_sV^*_s}[\sigma_{x_r}]_{ss'}\}\delta f_m(\hat{k}').
\end{aligned}
\tag{14}
$$

$$t_{MM} = t_{M'M'} = \cos(\theta/2),$$

$$t_{MM'} = t_{M'M} = iA \sin\theta \sin(\theta/2),$$

$$t_{Mm'} = t_{M'm} = 0,$$

$$t_{Mm} = t_{M'm'} = \frac{2k_M(1 - A\sin\theta)}{(k_M + k_m)}(\sin(\theta/2)),$$

$$r_{MM} = r_{M'M'} = \frac{k_M - k_m}{k_M + k_m}(1 - A\sin\theta)(1 - \cos\theta),$$

$$r_{MM'} = r_{M'M} = -i\frac{k_M - k_m}{k_M + k_m}A\sin^2\theta,$$

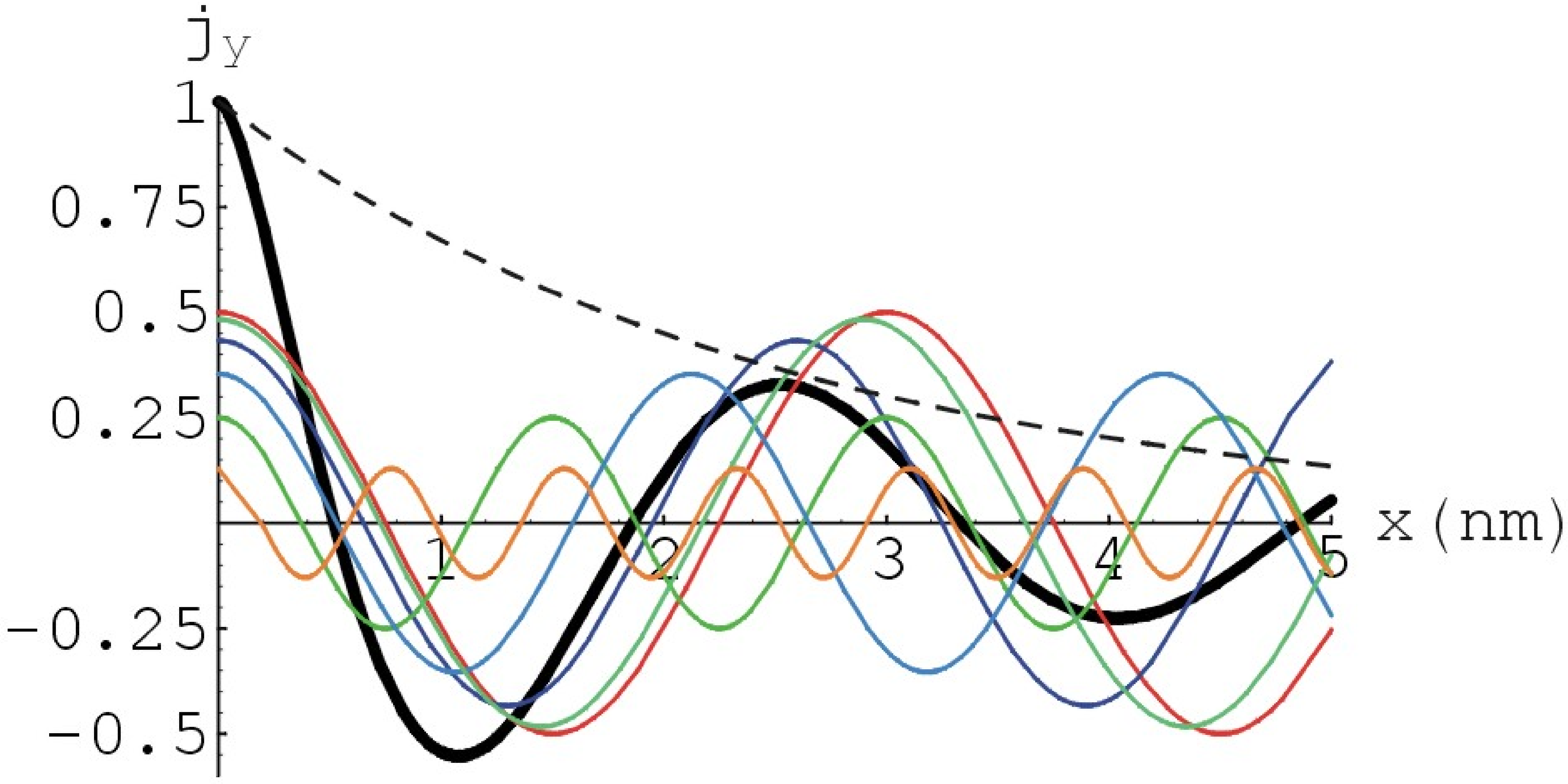
$$r_{Mm'} = r_{Mm} = r_{M'm'} = r_{M'm} = 0,$$

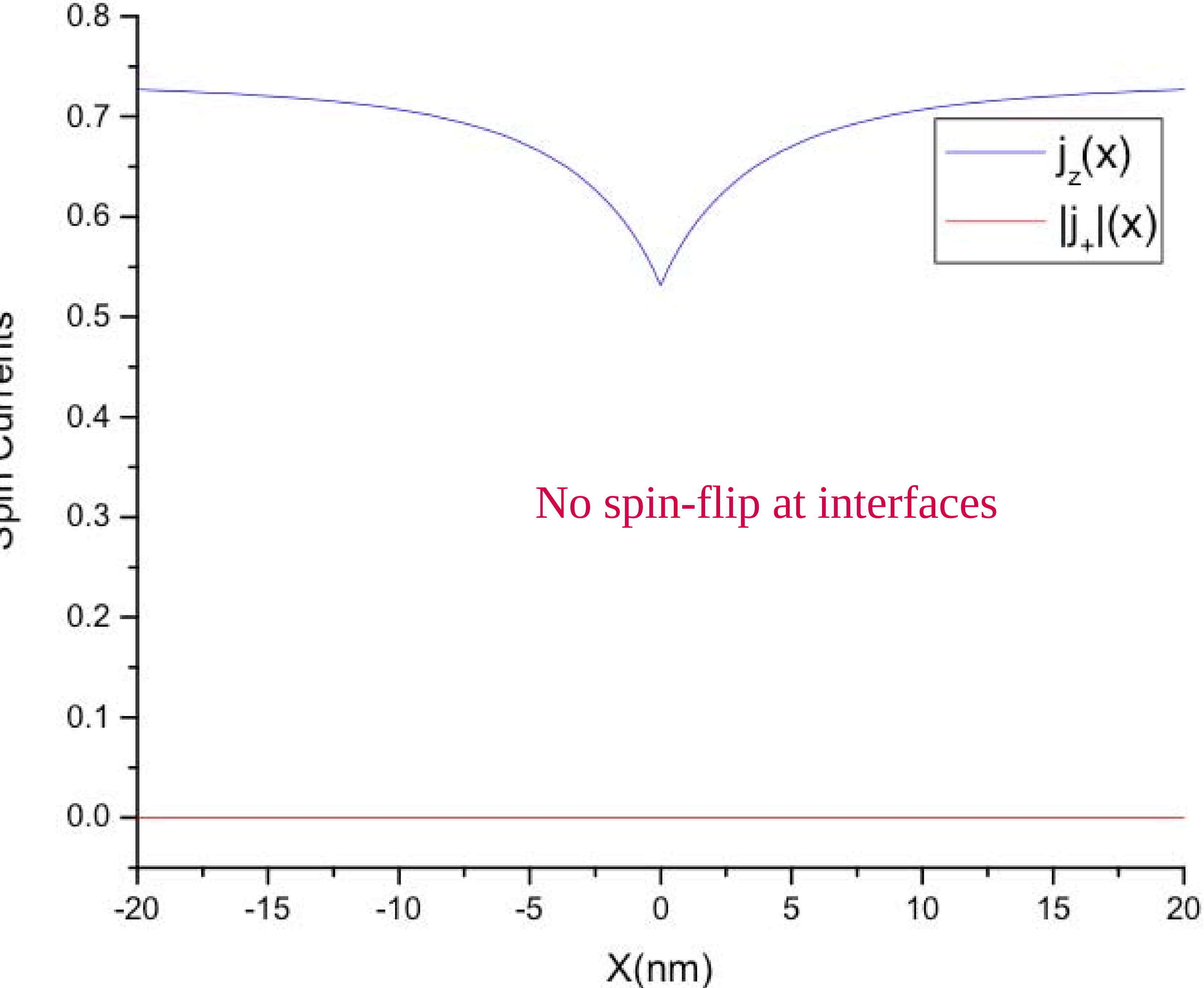
A is the new current induced spin-flip term

$$
\begin{aligned}
\mathbf{m}^{\pm}(x > 0) &= \int_{FS} d\mathbf{k} g_R^{\pm}(k, x > 0) \\
&= \int_{FS} d\mathbf{k} c_{\pm}^{>} \exp(\pm i \frac{J_k}{\hbar v_x} x).
\end{aligned}
$$

$$
\begin{aligned}
j^{\pm}(x > 0) &= \int_{FS} d\mathbf{k} v_x g_R^{\pm}(k, x > 0) \\
&= \int_{FS} d\mathbf{k} c_{\pm}^{>}(\mathbf{k}) v_x \exp(\pm i \frac{J_k}{\hbar v_x} x) \\
&\sim \int_{FS} d\mathbf{k} v_x \exp(\pm i \frac{J_k}{\hbar v_x} x),
\end{aligned}
$$

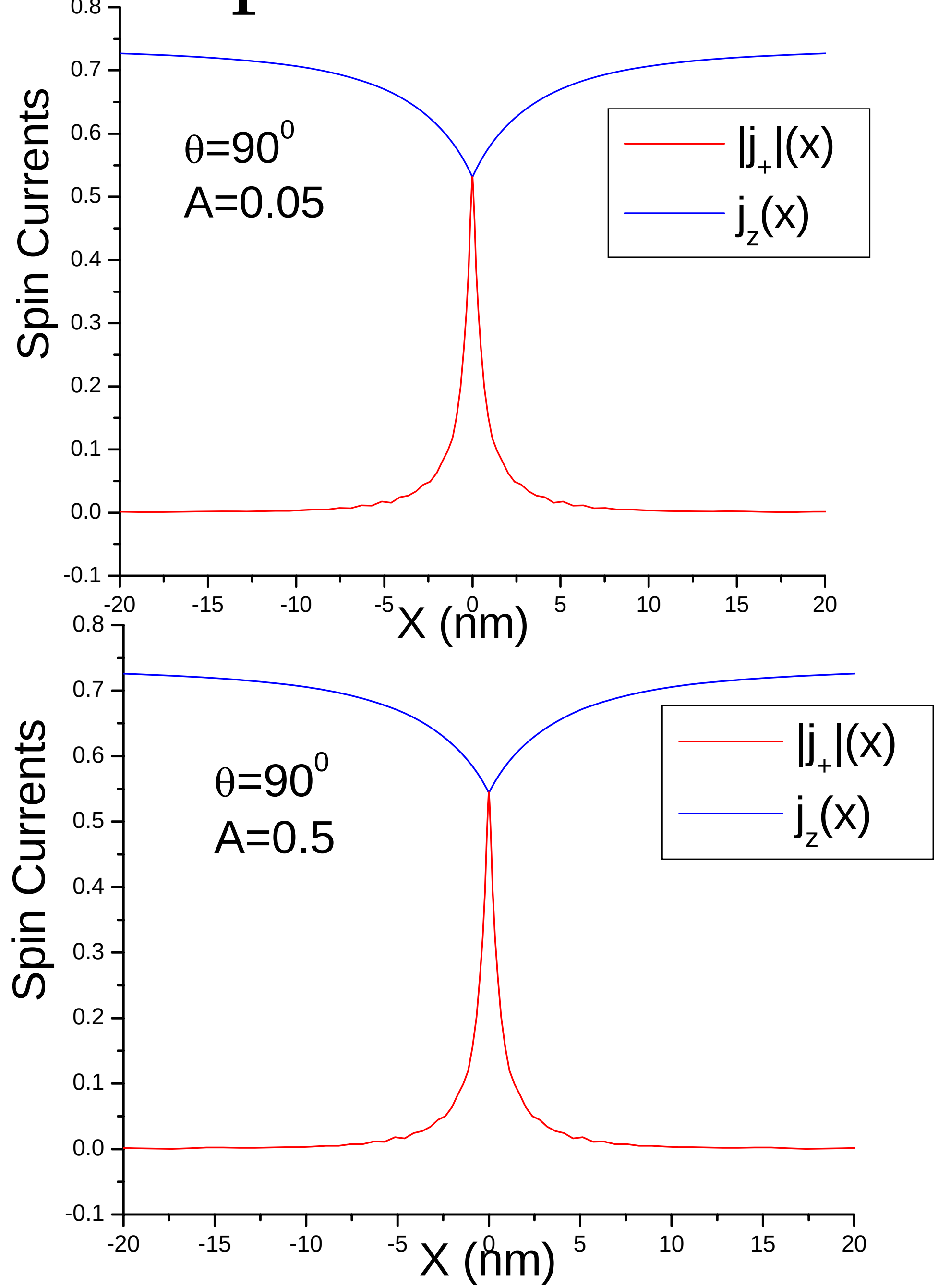
Interference pattern- y polarization



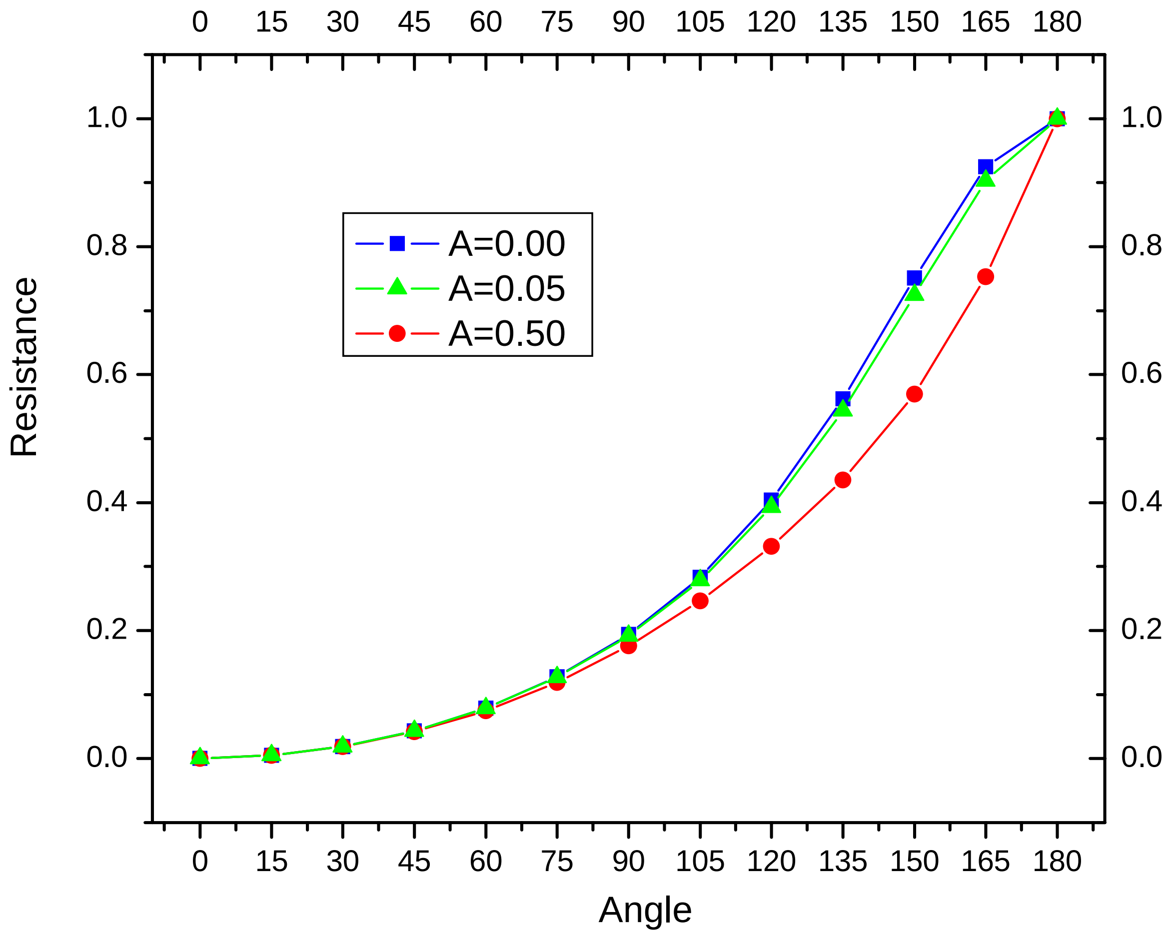


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Spin Currents
-20
-15
-10
-5
0
5
10
15
20
X(nm)
$j_z(x)$
$|j_+|(x)$
No spin-flip at interfaces

Spin currents

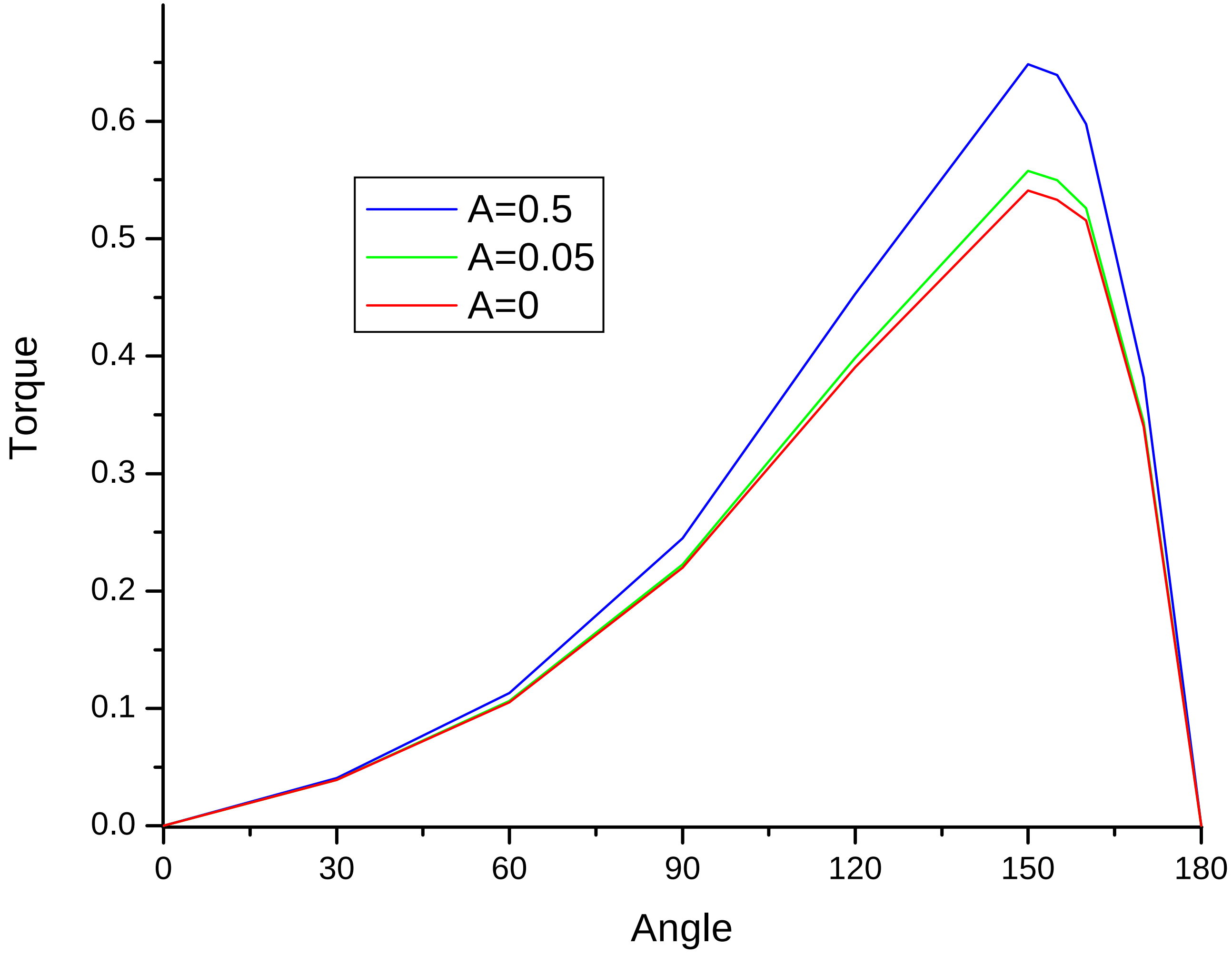


Resistance



0 15 30 45 60 75 90 105 120 135 150 165 180
1.0
0.8
0.6
0.4
0.2
0.0
Resistance
A=0.00
A=0.05
A=0.50
Angle

Spin torque as a function of angle between layers for three different cases of current induced spin flip (*CISP*)



Consequences

• Resistance is lower when one admits transverse currents in ferromagnetic layers.

•Angular variation of resistance and spin torque is changed upon including current induced spin flip, *CISP*, at interfaces.

•Spin torque is increased for same amount of energy expended when one includes *CISP*.

•True "mixing" conductance with an effective field component, as well as torque.

•Spatial variation of spin torque and effective field very different.

•*Observatio*n: Transmission from *Cu* to *Co* favors majority channel; penalizes minority channel conduction.

Time dependence of spin transport

$$\frac{\partial \vec{m}}{\partial t} + \frac{\partial \vec{j}_m}{\partial x} + (J/\hbar)\vec{m} \times \vec{M} = -\frac{\vec{m}}{\tau_{sf}},$$

$$\vec{j}_m = (\mu_B/e)p\vec{M}j_e - \xi\frac{\partial \vec{m}}{\partial x},$$

$$\frac{\partial \vec{m}}{\partial t} - \xi\frac{\partial^2 \vec{m}}{\partial x^2} + \frac{\vec{m}}{\tau_{sf}} + \frac{\vec{m} \times \vec{M}}{\hbar/J} = -(\mu_B/e)j_e\frac{\partial (p\vec{M})}{\partial x}.$$

Solution is found across entire multilayer by using source terms at interfaces. This obviates any assumptions about the scattering at interfaces; they are built into the Hamiltonian.

The source term in the global frame only has the Y direction component.

$$\frac{\partial(p\vec{M})}{\partial x} = \frac{\partial(p\vec{M})_y}{\partial x}$$

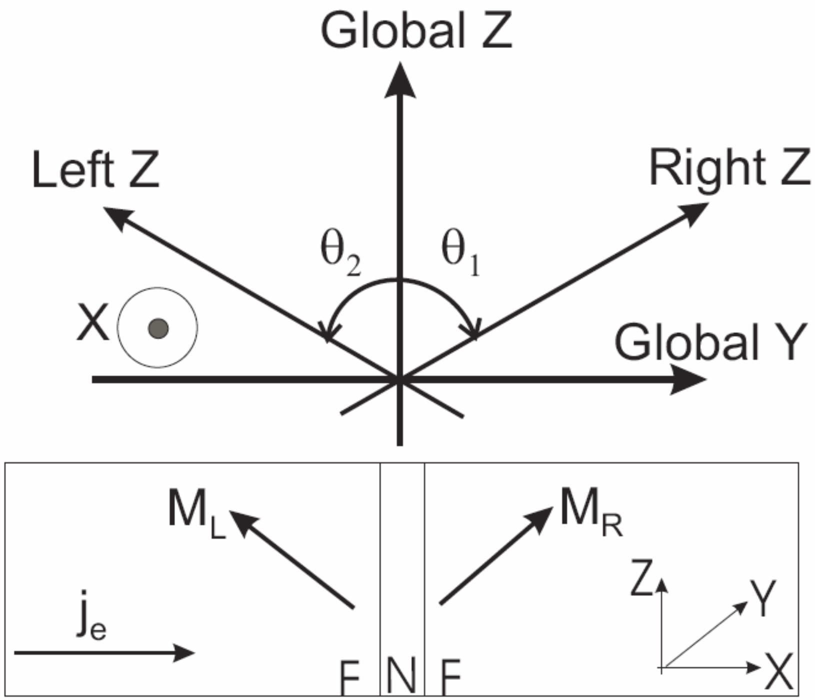
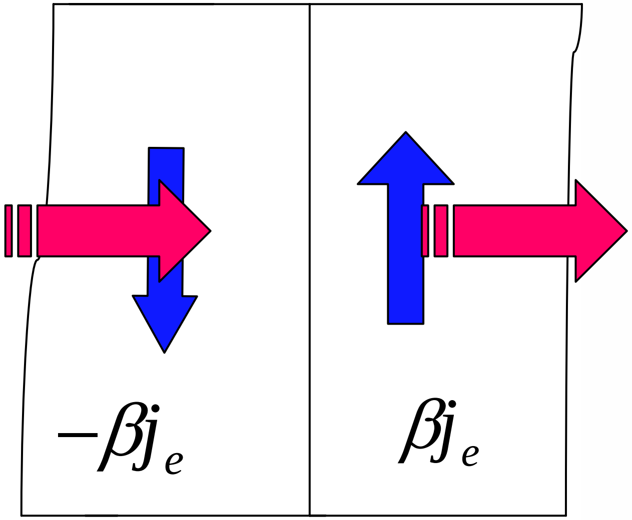
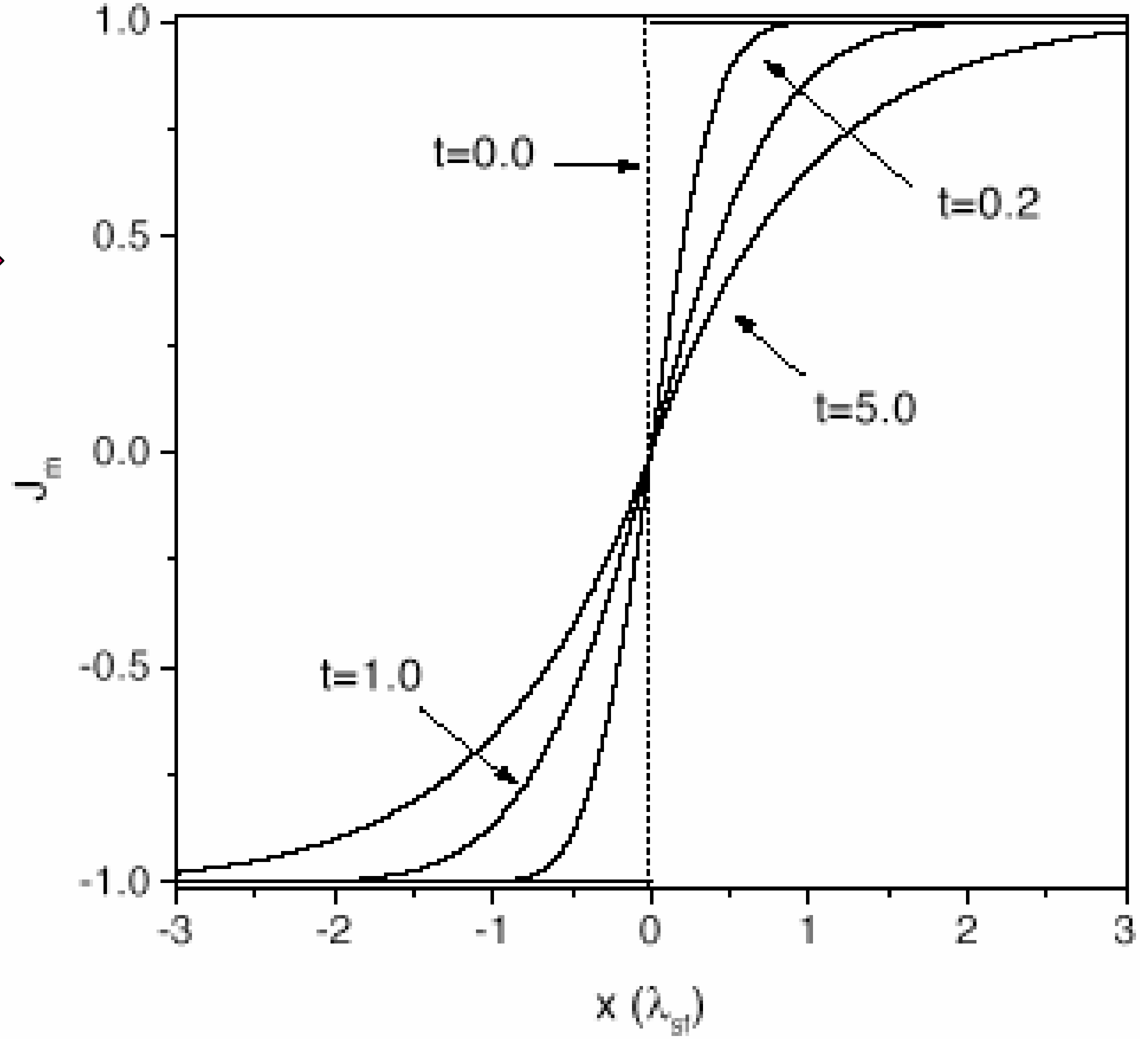


FIG. 1: Coordinates. The current is in x or growth direction in this multilayer. The magnetization directions of the two ferromagnetic layers M_L and M_R are in y-z plane. The global z direction is chosen in between M_L and M_R.



Spin currents

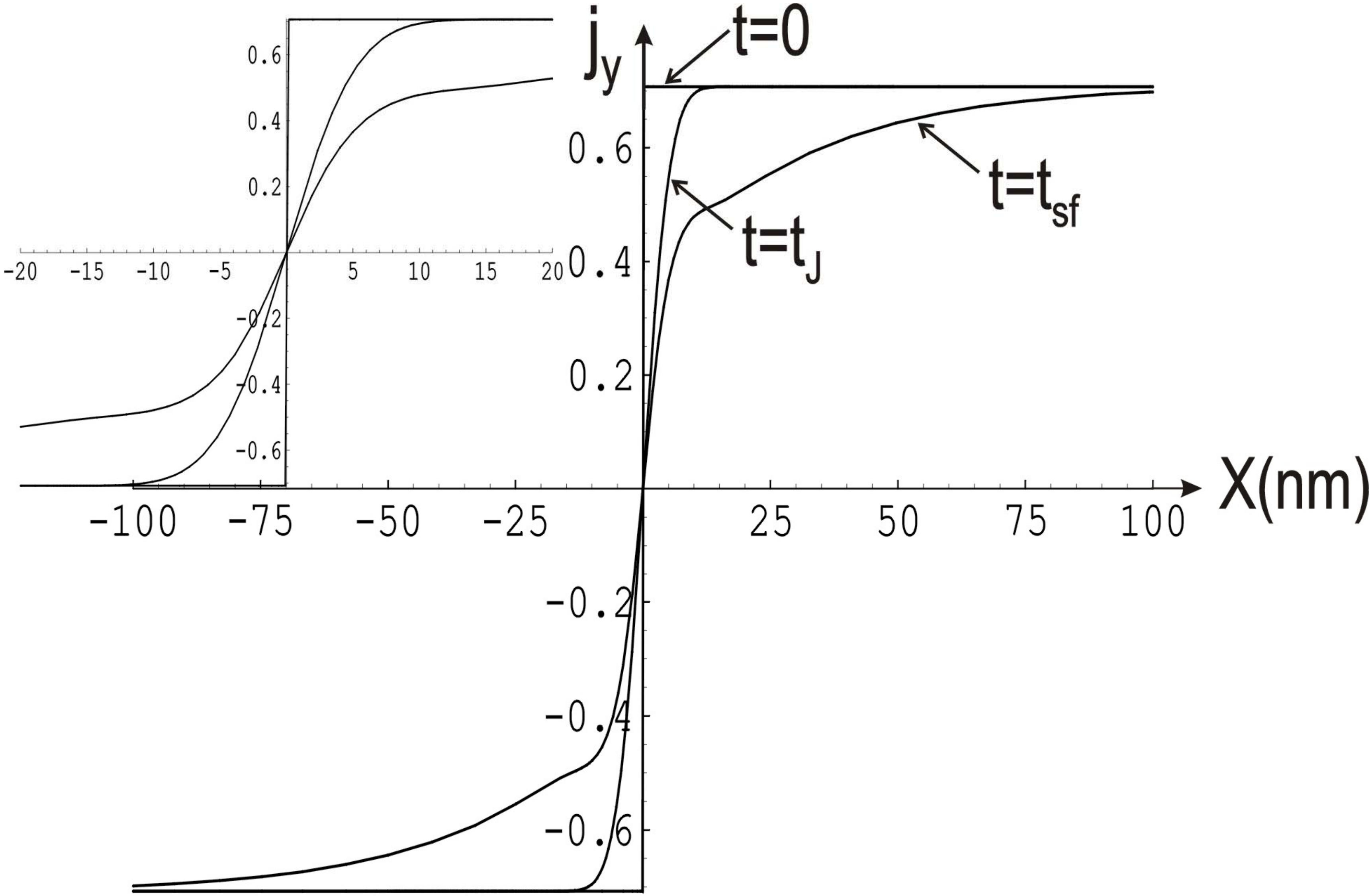


The magnetization current normalized to $(\mu_B / e)PJ_e$ as a function of position at times $t = 0.2\tau_{sf}, \tau_{sf}$, and $5\tau_{sf}$.

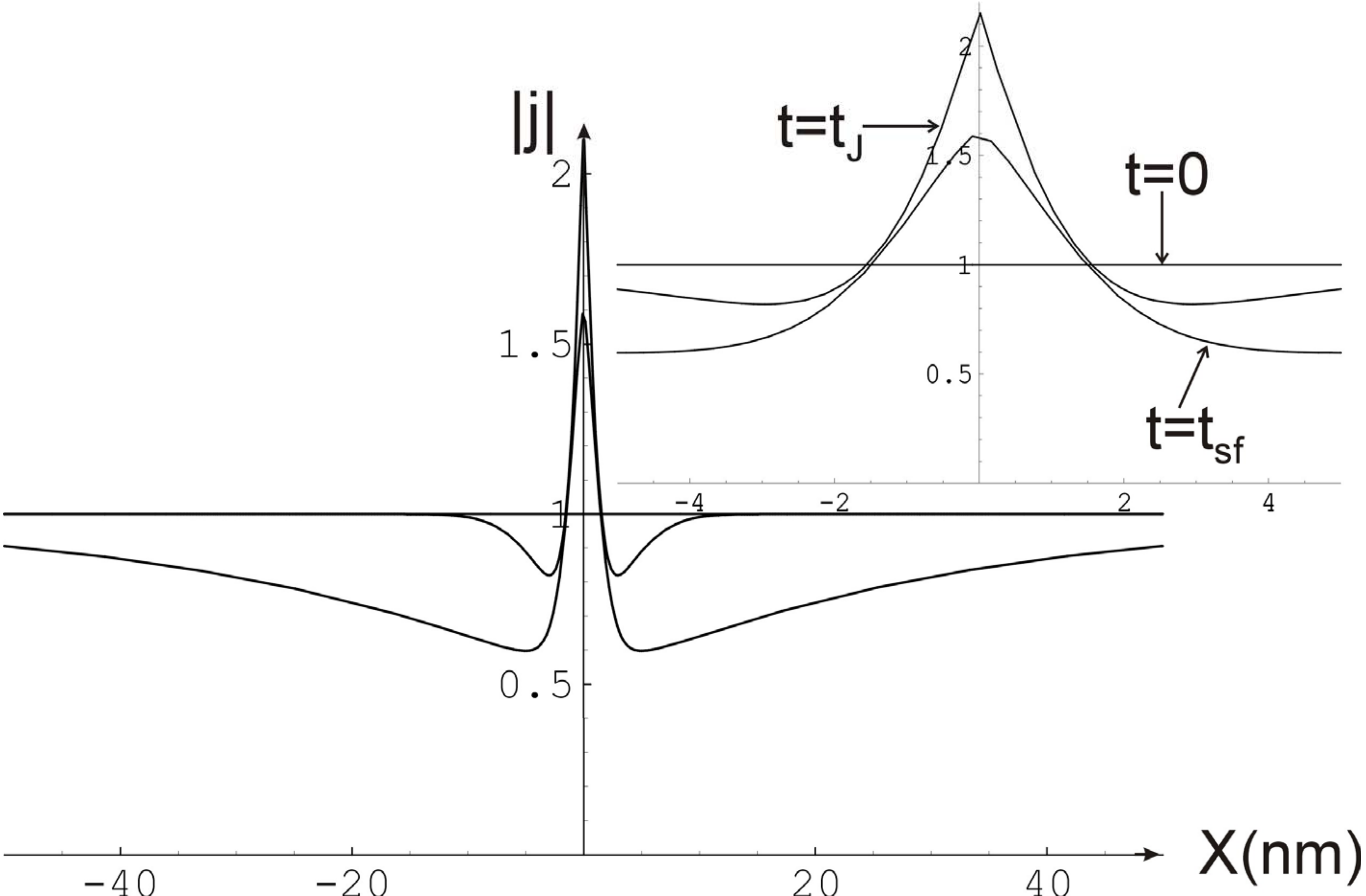
From: S. Zhang and P.M. Levy, Phys. Rev. B**65**, 052409 (2002).

Time evolution of spin current for layers 90^0 apart

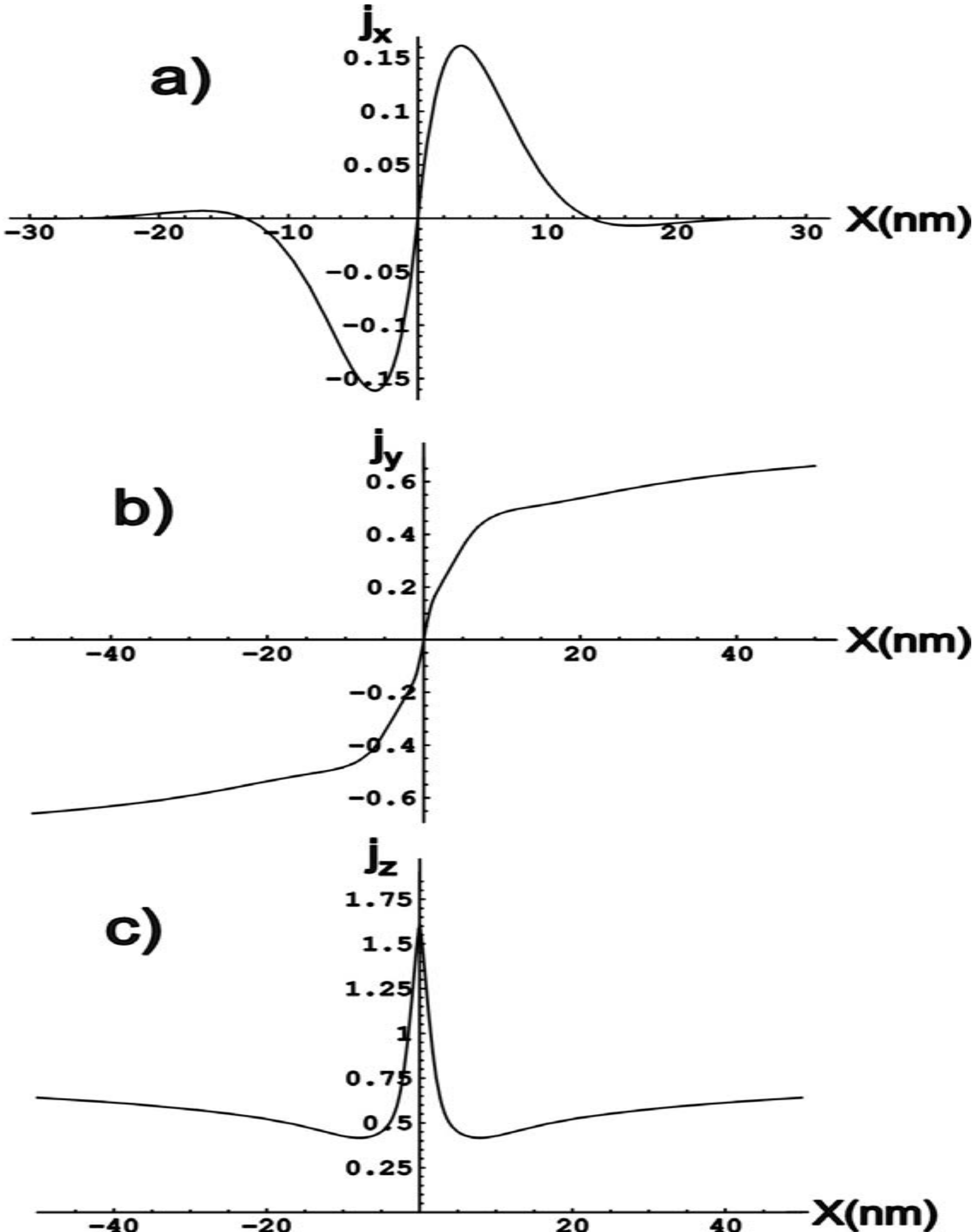
Components referred to global axes



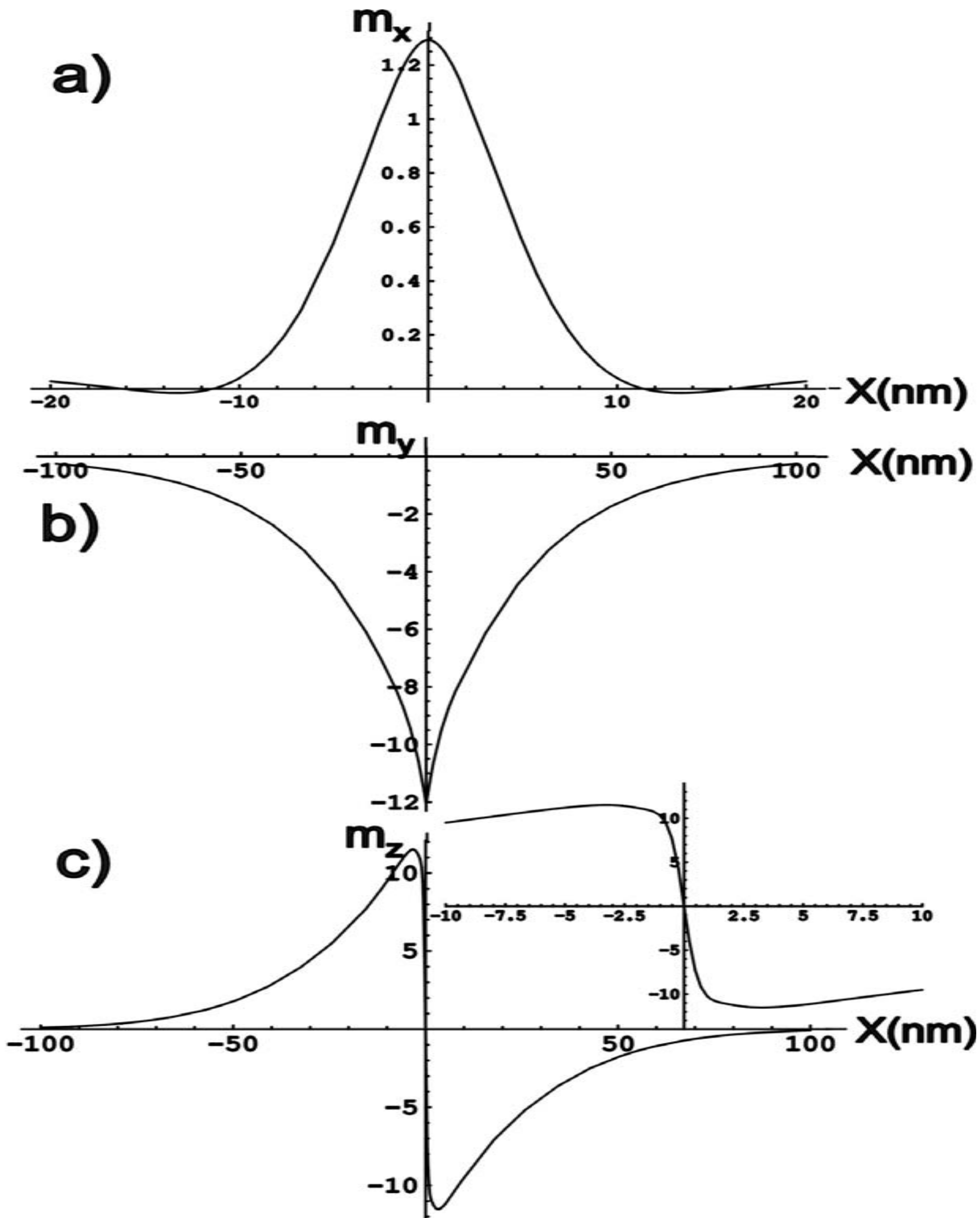
Time evolution of spin current for layers 90^0 apart
Components referred to global axes



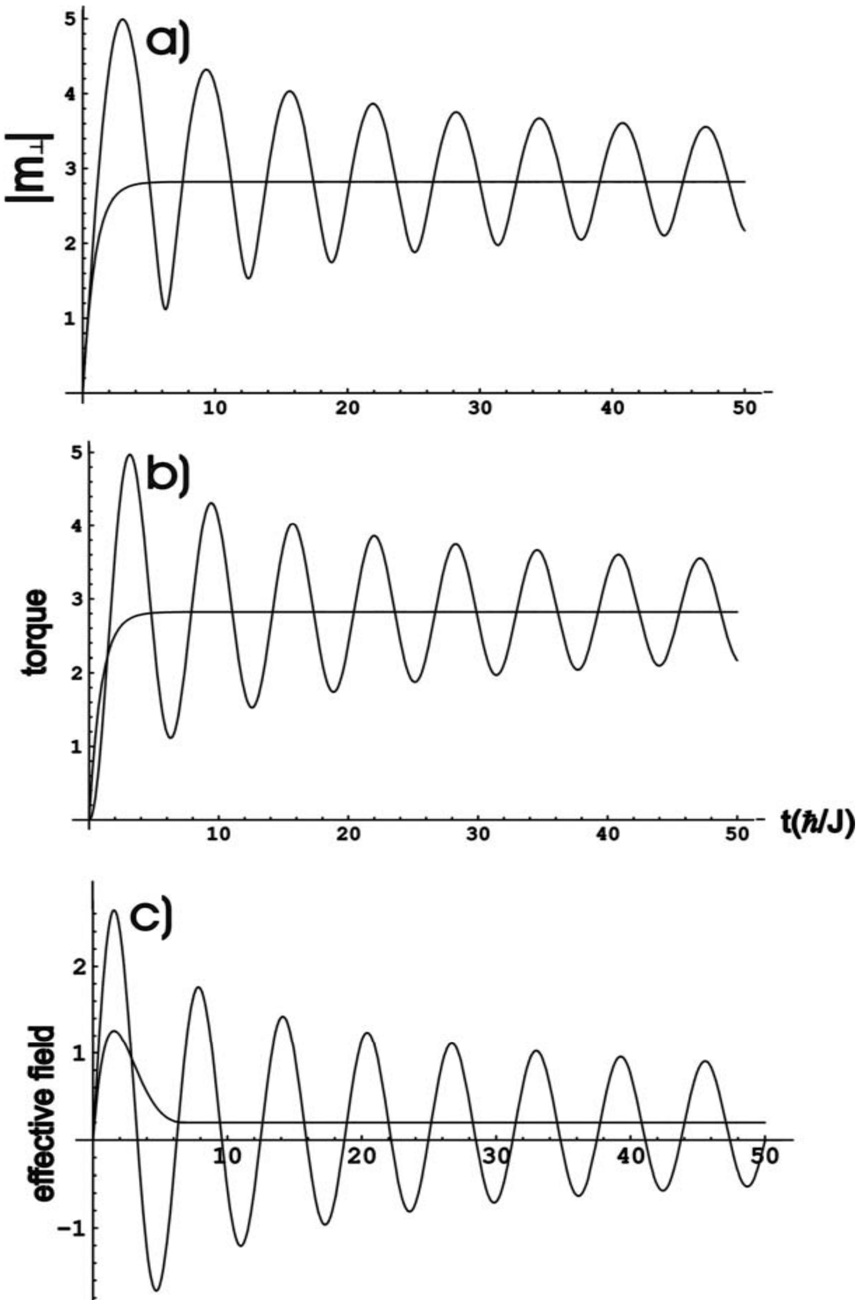
Spin currents at steady state



Spin accumulations at steady state



Time dependence of spin torque

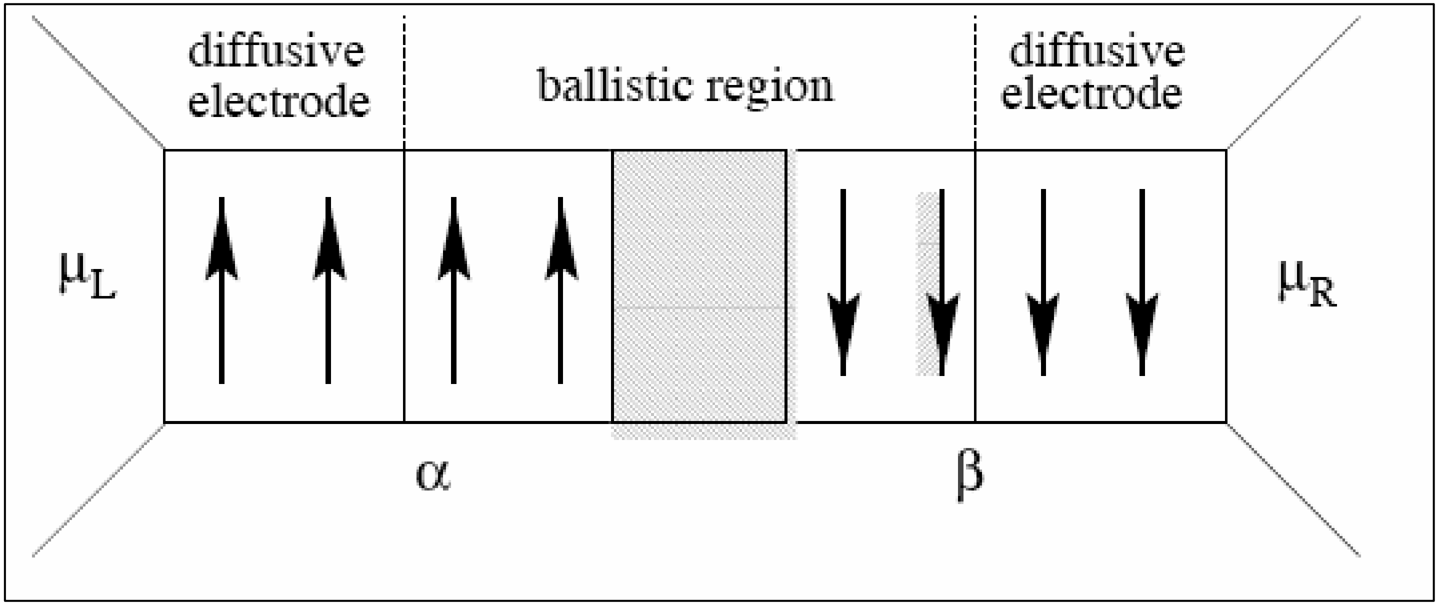


Ballistic transport: see S. Datta *Electronic Transport in Mesoscopic Systems (Cambridge Univ. Press, 1995).*

Collisionless regime; transport conditions set by reservoirs at boundaries. Conductance measured by transmission through states on Fermi surface $T_{k\sigma \to k'\sigma'} \propto \left| t_{k'\sigma',k\sigma} \right|^2$ in units of the quantum of conduction $2e^2/h = 12.9k\Omega^{-1}$

$G = \dfrac{2e^2}{h} MT,$ where M is the number of channels.

Critique of the "mantra" of Landauer's formula; see M.P. Das and F. Green, cond-mat/0304573 v1 25Apr 2003.



Particle current:

$$I_p = \frac{2e}{h}\left[T_{\beta\leftarrow\alpha}\mu_\alpha - T_{\alpha\leftarrow\beta}\mu_\beta\right]$$

$$\mu_\alpha \approx \mu_L \qquad \mu_\beta \approx \mu_R$$

$$T_{\beta\leftarrow\alpha} \propto \rho_\alpha\left(t_{\alpha\beta}\right)^* \rho_\beta\left(t_{\beta\alpha}\right)$$

Density matrix:

$$\hat{\rho} = \begin{pmatrix} \rho_\uparrow & 0 \\ 0 & \rho_\downarrow \end{pmatrix}$$

Rotated:

$$\Rightarrow \begin{pmatrix} \rho_0 + \rho_z \cos\theta & -i\rho_z \sin\theta \\ i\rho_z \sin\theta & \rho_0 - \rho_z \cos\theta \end{pmatrix}$$

Transmission amplitude:

$$\hat{t}_{\beta\alpha} = \begin{pmatrix} t_d + t_m S_z^{\alpha/\beta} & t_m S_-^{\alpha/\beta} \\ t_m S_+^{\alpha/\beta} & t_d - t_m S_z^{\alpha/\beta} \end{pmatrix}$$

$$\hat{t}_{\beta\alpha} = t_d \hat{1} + t_m \vec{\sigma} \cdot \vec{S}^{\alpha/\beta}$$

Charge current

$$T_\alpha \equiv Tr_\sigma \overset{?}{T}_{\beta\leftarrow\alpha} = T_\beta \equiv Tr_\sigma \overset{?}{T}_{\alpha\leftarrow\beta}$$

$$I_c = \frac{2e^2 V}{h} Tr_\sigma \overset{?}{T}$$

Spin current

$$Tr_\sigma \vec{\sigma} \overset{?}{T}_{\alpha\leftarrow\beta} \neq Tr_\sigma \vec{\sigma} \overset{?}{T}_{\beta\leftarrow\alpha}$$

$$\vec{T}_\alpha \equiv Tr_\sigma [\vec{\sigma} \overset{?}{T}_{\beta\leftarrow\alpha}]$$

$$\vec{T}_\beta \equiv Tr_\sigma [\vec{\sigma} \overset{?}{T}_{\alpha\leftarrow\beta}]$$

$$\vec{I}_s = \frac{2e}{h}\left[\vec{T}_\alpha \mu_\alpha - \vec{T}_\beta \mu_\beta\right]$$

$$= \frac{2e}{h}\left\{ {}^1\!/_2 \left(\mu_\alpha + \mu_\beta\right)\left[\vec{T}_\alpha - \vec{T}_\beta\right] + eV \cdot {}^1\!/_2 \left[\vec{T}_\alpha + \vec{T}_\beta\right]\right\}$$

Inelastic scattering; let's confine ourselves to T=0K:

Only possible to generate magnons when they are emitted by spin current.

$$\mu_{\alpha/\beta} \Rightarrow \mu_{\alpha/\beta} - \hbar\omega_q^{\alpha/\beta}\Theta(eV - \hbar\omega_q^{\alpha/\beta})$$

$$\vec{I}_s = \frac{2e}{h}\left[\vec{T}_\alpha \mu_\alpha - \vec{T}_\beta \mu_\beta\right]$$

$$\vec{I}_s^{magnon} = -\frac{2e}{h}\sum_q \hbar\omega_q^{\alpha/\beta}\Theta(eV - \hbar\omega_q^{\alpha/\beta})\left[\vec{T}_\alpha - \vec{T}_\beta\right]$$

Evaluation of sum over magnons

$$\sum_q \hbar\omega_q^{\alpha/\beta}\Theta(eV-\hbar\omega_q^{\alpha/\beta}) = \int_0^{eV} d\omega g^{\alpha/\beta}(\omega)\hbar\omega$$

Interfacial magnons

$$\vec{I}_s^{\,magnon} = -\frac{eN^i_{\alpha/\beta}}{h}\left(\frac{eV}{E_m^{\alpha/\beta}}\right)(eV)\hbar\left[\vec{T}^i{}_\alpha - \vec{T}^i{}_\beta\right]$$

where superscript i stands for transmission amplitudes for interface magnon production t_m^i.

Remember the spin current due to elastic scattering is:

$$\vec{I}_s = \frac{e}{h}eV\cdot\left[\vec{T}_\alpha + \vec{T}_\beta\right]$$

For $t_m=0$

$$\hat{t}_{\beta\alpha} = t_d \hat{1} + t_m \vec{\sigma}\cdot\vec{S}^{\alpha/\beta} \quad \Rightarrow \quad \hat{T}_{\beta\leftarrow\alpha} = \left|t_d\right|^2 \hat{\rho}_\alpha \hat{\rho}_\beta$$

$$(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b} + i\vec{\sigma}\cdot(\vec{a}\times\vec{b})$$

Equilibrium spin current

$$\left[\vec{T}_\alpha - \vec{T}_\beta\right] \propto \vec{\rho}_\alpha \times \vec{\rho}_\beta$$

None other than interlayer exchange coupling

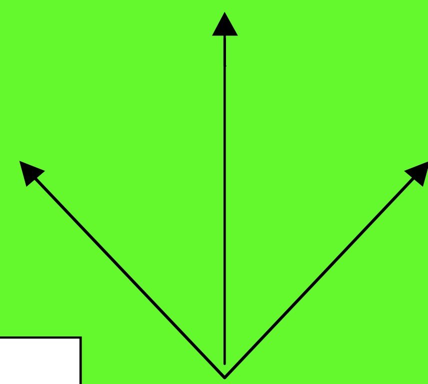
Out of equilibrium spin current

$$\left[\vec{T}_\alpha + \vec{T}_\beta\right] \propto \rho_0^\beta \vec{\rho}_\alpha + \rho_0^\alpha \vec{\rho}_\beta$$

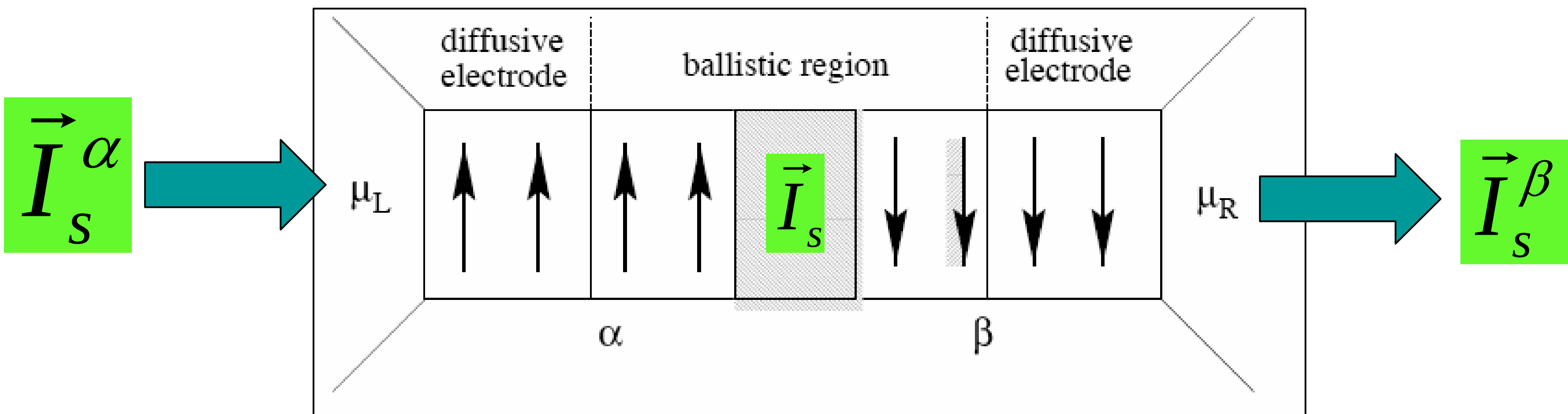
Spin current

$$\vec{I}_s \sim \frac{1}{2}\left[\vec{I}_s^{\,\alpha} + \vec{I}_s^{\,\beta}\right]$$

$$\vec{I}_s = \frac{e}{h} eV \cdot \left[\vec{T}_\alpha + \vec{T}_\beta\right]$$



$\vec{I}_s^{\,\alpha}$ $\vec{I}_s^{\,\beta}$



Torque on an electrode

$$\vec{\tau}^{\alpha} \equiv -\hbar(\vec{I}_s - \vec{I}_s^{\alpha})$$

$$\vec{\tau}^{\beta} \equiv -\hbar(\vec{I}_s^{\beta} - \vec{I}_s)$$

$$\vec{\tau}_{\perp}^{\alpha} = -\hbar\left[(\vec{I}_s - \vec{I}_s^{\alpha}) \times \vec{\alpha}\right] \times \vec{\alpha} = -\hbar\left(\vec{I}_s \times \vec{\alpha}\right) \times \vec{\alpha}$$

$$\vec{\tau}_{\perp}^{\beta} = -\hbar\left[(\vec{I}_s^{\beta} - \vec{I}_s) \times \vec{\beta}\right] \times \vec{\beta} = \hbar\left(\vec{I}_s \times \vec{\beta}\right) \times \vec{\beta}$$

$$\tau_y^{\alpha} \propto eV\left|t_d\right|^2 \sin\vartheta \rho_0^{\alpha} \rho_z^{\beta}$$

$$\tau_y^{\beta} = \tau_y^{\alpha}(\alpha \leftrightarrow \beta)$$

The only current or bias induced excitations are from $\sigma_{\pm}$

and we have to evaluate

$$\left\langle S_{\pm}^{\alpha/\beta} S_{\mp}^{\alpha/\beta} \right\rangle \text{ at } T = 0K \Rightarrow 2S^{\alpha/\beta}\hbar^2$$

While for the *elastic* terms (non spin-flip magnetic as well as for direct transmission) we found:

$$\tau_y^{\beta} = \tau_y^{\alpha}(\alpha \leftrightarrow \beta)$$

The new feature for the inelastic contributions to the torque are that they are not in the same direction for the two electrodes:

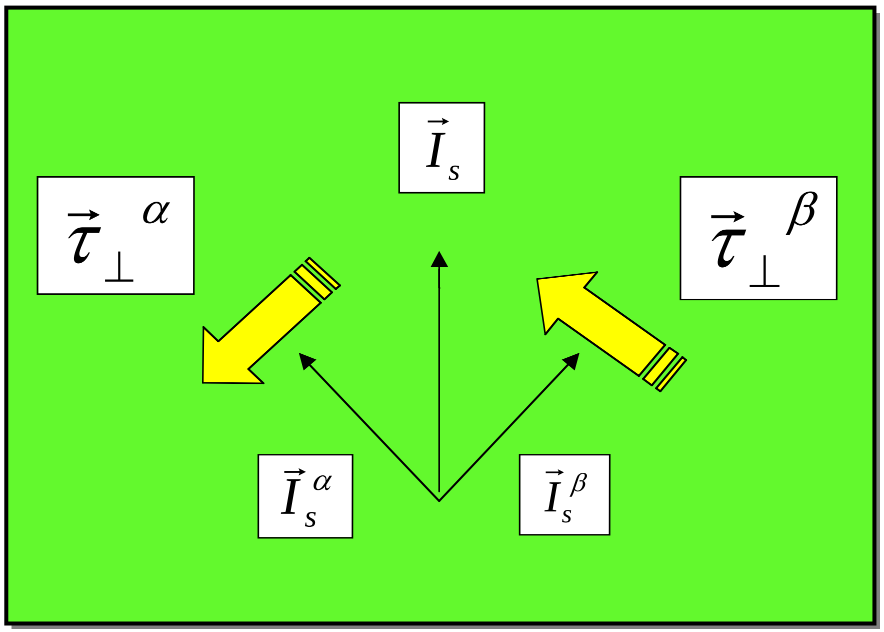
$$\tau_y^{\beta} = -\tau_y^{\alpha}(\alpha \leftrightarrow \beta)$$

Definition of spin torque:

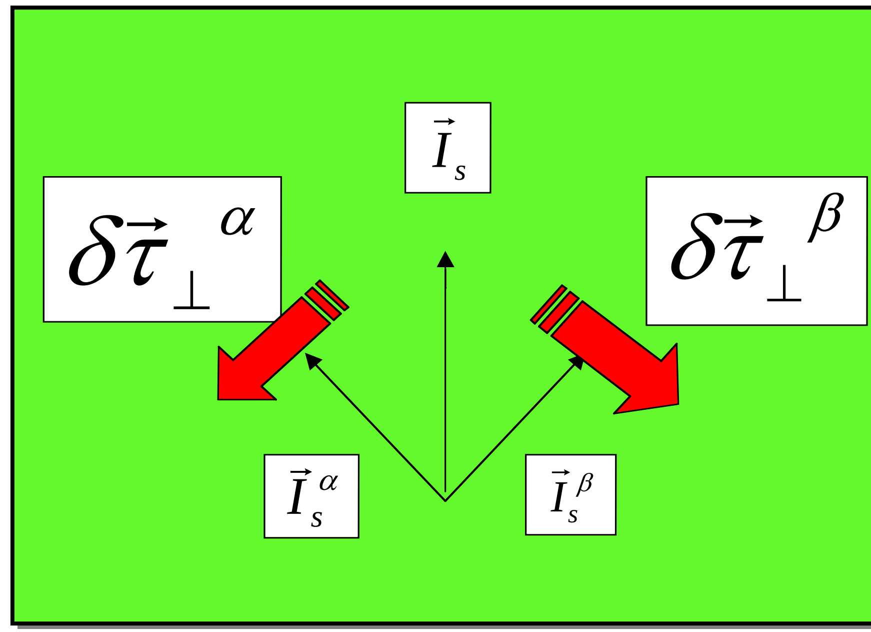
$$\vec{\tau}_{\perp}^{\alpha} = -\hbar\left(\vec{I}_s \times \hat{\alpha}\right)\times \hat{\alpha}$$

$$\vec{\tau}_{\perp}^{\beta} = \hbar\left(\vec{I}_s \times \hat{\beta}\right)\times \hat{\beta}$$

Elastic



Inelastic



Magnons created by hot spin current *assist* elastic torque on upstream electrode, but for downstream are in *opposite* sense.

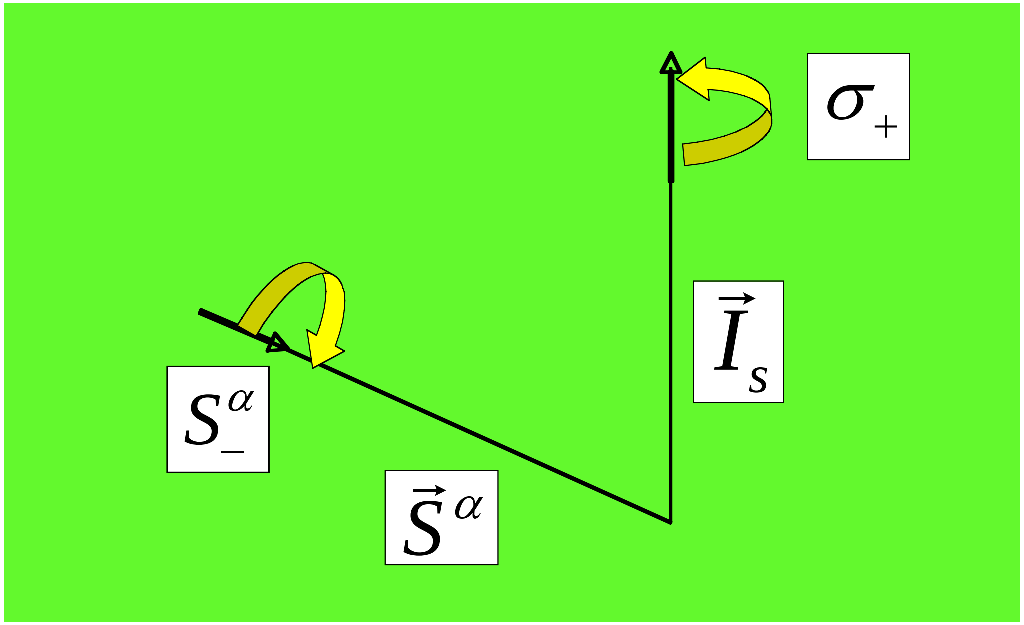
How does one understand this?

Elastic torque comes from spin current in tunnel junction being the vector sum of the polarized currents from the source and drain, i.e., from upstream and downstream electrodes.

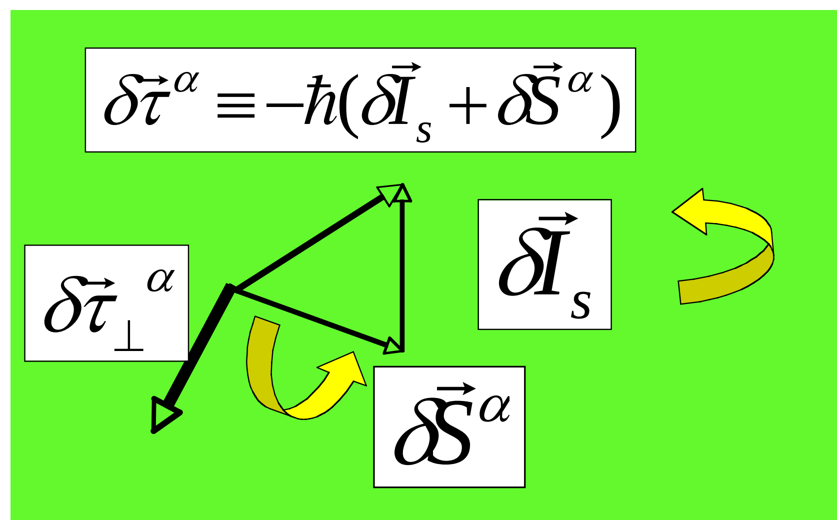
From our calculations we find

When angular momentum is transferred between a spin current whose polarization is noncollinear to the magnetization of an electrode, torque is produced. The component of the *vector* sum of difference between spin angular momentum gained by current and that lost by background magnetization that is transverse to electrode's magnetization is the torque created by this exchange of magnons between noncollinear entities.

At T=0K hot spin currents can only lower the polarization of electrodes.



Note the plus sign in definition of torque due to transfer of angular momentum



Summarizing:

$$\vec{I}_s = \frac{2e}{h}\left\{\left[\vec{T}_\alpha \mu_\alpha - \vec{T}_\beta \mu_\beta\right] - \sum_q \hbar\omega_q^{\alpha/\beta}\,\Theta(eV - \hbar\omega_q^{\alpha/\beta})\left[\vec{T}_\alpha - \vec{T}_\beta\right]\right\}$$

$$= \frac{2e}{h}\left[\vec{T}_\alpha - \vec{T}_\beta\right]\left\{\tfrac{1}{2}\left(\mu_\alpha + \mu_\beta\right) - \sum_{i=\alpha,\beta}\sum_q \hbar\omega_q^i\,\Theta(eV - \hbar\omega_q^i)\right\}$$

$$+ \frac{2e}{h}\left[\vec{T}_\alpha + \vec{T}_\beta\right]\cdot \tfrac{1}{2}\left(\mu_\alpha - \mu_\beta\right)$$

$$\vec{\tau}_\perp^{\,\alpha} = -\hbar\left(\vec{I}_s \times \hat{\alpha}\right)\times\hat{\alpha} \qquad \vec{\tau}_\perp^{\,\beta} = \hbar\left(\vec{I}_s \times \hat{\beta}\right)\times\hat{\beta}$$

$\vec{\tau}_\perp^{\,\alpha}\Big|_{\text{elastic}}$ in same direction as $\vec{\tau}_\perp^{\,\beta}\Big|_{elastic}$

$\vec{\tau}_\perp^{\,\alpha}\Big|_{\text{inelastic}}$ in same direction as $\vec{\tau}_\perp^{\,\alpha}\Big|_{elastic}$

$\vec{\tau}_\perp^{\,\beta}\Big|_{\text{inelastic}}$ in *opposite* direction as $\vec{\tau}_\perp^{\,\beta}\Big|_{elastic}$

That's all for today folks

Rotating one magnetic layer relative to another produces new effects.

Due to *non*collinearity of background magnetization, polarization of spin current at an angle with respect to local background. Transverse component of spin current.

$$\nabla \cdot \mathbf{j}_m \neq 0 \Rightarrow \frac{\partial \mathbf{m}}{\partial t} \neq 0$$

To achieve steady state additional spin accumulation transverse local background magnetization. Distance over which accumulation is not parallel to background $\lambda_J << \lambda_{sdl}$.

Over distance where transverse exists, exchange interaction between spin accumulation and magnetic background creates a torque on latter.

Introduction of noncollinearity creates a torque on background which acts in such a way as to undo it.

Rotation is such as to restore collinear structure, this restores $\nabla \cdot \mathbf{j}_m = 0$, at least over distances less than λ_{sdl}.

On time scales long compared to that for the conduction electron spins, $t \approx 10^{-9}$ sec, background rotates due to torque created by spin currents through the transverse spin accumulation.

Therefore, for electron transport phenomena we can assume the magnetic background remains stationary - the adiabatic approximation